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Saturated PI control of continuous bioreactors with Haldane kinetics ☆

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ABSTRACT

The problem of designing a saturated Output-Feedback (OF) controller for a continuous bioreactor with Haldane kinetics is addressed. The reactor must be operated at maximum biomass production rate by manipulating the feed rate on the basis of a (biomass or substrate) measurement. The consideration of the problem as an interlaced observer-control design leads to a saturated PI controller that recovers (up to observer convergence) the behavior of a detailed model-based robust nonlinear state-feedback (SF) globally stabilizing saturated controller. The saturated PI control scheme has: (i) closed-loop stability conditions in terms of control gains and limits, and (ii) a simple construction-tuning procedure. The proposed approach is illustrated and tested with a representative case example through numerical simulations.

1. Introduction

The development of Output-Feedback (OF) control schemes for bioreactors is motivated by the need of designing or redesigning automated operations for improved behavior, with better compromises between product quality, productivity, safety and costs (Henson, 2006). Bioreactors with nonlinear Haldane kinetics exhibit typical nonlinear behavior, including multiplicity, parametric sensitivity, asymmetric input-state response, limit cycling and so on (Di Bassio et al., 1978; Zhang and Henson, 2001). Consequently, the associated joint process-control design problem is an interesting and complex task, especially when simplicity, reliability and low development-maintenance costs are important application-oriented design specifications.

In the industrial practice (Henson, 2006) conventional PI and PID control schemes are widely employed. These controllers yield good performance with low development, implementation and maintenance costs, and without needing a detailed process model. However, the application of the PI controllers to systems with complex nonlinear behavior requires special attention to closed-loop stability and control saturation. Motivated by these issues several studies on nonlinear upgrades of the conventional linear PI controller were proposed (Reddy and Chidambaram, 1994; Antonelli and Astolfi, 2000) but lacked a robust stability assessment for time varying perturbations. On the other hand, the problem of tuning of PI controllers was addressed using Gain Scheduling (see e.g. the discussion in Shimizu, 1993) and more recently using artificial intelligence (Satishkumar and Chidambaram, 1999; Kumar et al., 2008), showing good performance. In particular, the control saturation feature is a central issue in the design of a PI scheme for an open-loop bioreactor with non-monotonic kinetics where the maximum biomass production steady-state (SS) is non-unique, possibly structurally unstable, and can be accompanied by the undesired washout SS (Andrews, 1968; Antonelli and Astolfi, 2000). The problem of saturated PI control was addressed in Jadot et al. (1999) and stability conditions were drawn in terms of controller gains and limits. The related problem of antiwindup protection was addressed in Kapoor and Daoutidis (1999) and Doyle (1999) via observer-based antiwindup protection schemes with local stability results.

Hitherto, the PI control schemes for bioreactors lack: (i) systematic construction-tuning procedures, and (ii) robust closed-loop stability criteria coupled with simple tuning guidelines and control limit designs. The majority of the advanced OF control studies have been performed for the unconstrained control case, and the development of the considerably more complex OF saturated control lags far behind.

In the chemical reactor engineering field, the State-Feedback (SF) saturated control problem has been addressed: (i) by *ad hoc*

☆ Preliminary results of this paper for the unconstrained OF control design problem were presented at the Ninth IFAC International Symposium on Dynamics and Control of Process Systems (DYCOPS 2010), Leuven, Belgium, July 5–7, 2010: (Schaum et al., 2010).

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devising saturation handling schemes for nonlinear geometric (Calvet and Arkun, 1998; Kendi and Doyle, 1998), and geometric-MPC (Kurtz and Henson, 1997) SF controllers, and (ii) with rigorous closed-loop stability assessments for nonlinear geometric SF controllers (Chen and Chang, 1985; Alvarez et al., 1991; Alonso and Banga, 1998; Kapoor and Daoutidis, 2000). The OF saturated control problem has been addressed with rigorous stability treatments: (i) Chen and Chang (1985) drew local stability conditions for P and PI controllers, and (ii) El-Farra and Christofides (2003) drew nonlocal stability conditions for a class of minimum phase nonlinear systems with well-defined relative degree, using a combination of a nonlinear geometric SF controller with a high-gain saturated nonlinear observer following the ideas of Teel and Praly (1995) who analyzed the advantages of saturation on observer design. In particular, based on the inverse optimal approach (Freeman and Kokotovic, 1996; Sepulchre et al., 1997), in El-Farra and Christofides (2003) and Diaz-Salgado et al. (2011) a theoretically based-connection between optimality and robustness was included.

In the bioreactor engineering field Barron and Aguilar (1998) drew local stability conditions for a nonlinear geometric SF and a PI OF saturated controller for Monod (monotonic) kinetics. Rapaport and Harmand (2002) drew local stability conditions for an OF controller built as the combination of a nonlinear geometric SF controller with an interval observer for Haldane (non-monotonic) kinetics. Mendez-Acosta et al. (2005) obtained practical stability for an unconstrained OF controller made by the interconnection of a nonlinear geometric SF controller with a state-input high-gain nonlinear observer for Monod and Haldane kinetics, and performed in an ad hoc manner (through simulation testing) an antiwindup scheme-based OF saturated control implementation.

Summarizing, even though valuable insight on the saturated control problem for exothermic reactors and bioreactors has been gained, the problem is still open. The powerful advanced nonlinear control approach offers rigorous theoretical backup and systematization, but yield complex, strongly model-dependent nonlinear dynamical control schemes. The *ad hoc* implementation of saturated PI controllers lacks robust stability conditions implying a lack of reliability.

These considerations motivate of the scope of the present study: the development of an application-oriented saturated OF controller design for a continuous bioreactor with Haldane kinetics within an advanced nonlinear dynamics-control framework. The reactor must be operated at maximum biomass production rate by manipulating the feed rate on the basis of a (biomass or substrate) measurement. We are interested in designing an OF control scheme with: (i) saturation handling capabilities, (ii) simple implementation and systematic design, (iii) robust stability assessment coupled with simple tuning guidelines, (iv) reduced model dependency, and (v) control limit design. The result is a near-to-optimal PI saturated controller with implicit anti-windup protection scheme, which represents an upgrade with respect to conventional PI saturated control schemes. Methodologically speaking, we are interested in putting the proposed approach in perspective with the theoretical connections (Sepulchre et al., 1997) between robustness, optimality, and passivity which underlie the robust chemical process constructive control approach (El-Farra and Christofides, 2003; Gonzalez and Alvarez, 2005), and with conventional PI control schemes with anti-windup protection (Kothare et al., 1994; Jadot et al., 1999). The results obtained represent an extension to the saturated case of the feedforward (FF)-OF control design employed in chemical reactors (Gonzalez and Alvarez, 2005; Alvarez and Gonzalez, 2007; Diaz-Salgado et al., 2011), distillation columns (Castellanos-Sahagun and Alvarez, 2006), and biological reactors (Schaum et al., 2010). The proposed approach is illustrated and tested with a representative example through simulations.

2. Control problem

Consider a continuous stirred tank bioreactor with volume V and Haldane (inhibited) kinetics $K(S)$, where substrate is fed at rate q and concentration S_e, and converted into biomass at concentration B. Without restricting the approach, the case of biomass measurement ($Y=B$) will be considered, in the understanding that the cases of substrate and reaction rate measurements can be addressed with the same methodology. The reactor dynamics are given by the mass balances (Bailey and Ollis, 1986; Bastin and Dochain, 1990)

$$\frac{dS}{dt_a} = -\frac{1}{Y_S}K(S)B + D(S_e - S), \quad 0 < D^- \le D \le D^+, \quad S(0) = S_0,$$

$$\frac{dB}{dt_a} = K(S)B - DB, \quad B(0) = B_0, \quad P = DB, \quad Y = B, \tag{1}$$

where

$$K(S) = \frac{K_0 S}{S^2/K_i + S + K_s}, \tag{2}$$

S (or B) is the substrate (or biomass) concentration, t_a is the actual time, D is the dilution rate control input with low (or high) limit D^- (or D^+), $K(S)$ is the specific biomass growth rate with mass action (K_0), substrate inhibition (K_i), and substrate saturation (K_s) constants. The constant Y_S is the substrate-to-biomass yield coefficient, P is the biomass production rate, and Y is the biomass measurement.

For the purpose at hand, introduce the dimensionless variables

$$s = S/S_r, \quad b = B/S_r, \quad t = t_a D_r, \tag{3}$$

where (S_r, D_r) is a reference constant value pair, and rewrite the bioreactor (1) in the dimensionless form

$$\dot{s} = -\rho(s)b/\gamma + \theta(s_e - s), \quad s(0) = s_0,$$

$$\dot{b} = \rho(s)b - \theta b, \quad y = b, \quad p = \theta b, \quad b(0) = b_0, \tag{4}$$

where

$$\dot{(\,)} = d(\,)/dt, \quad \theta = D/D_r, \quad y = Y/S^0,$$

$$p = P/(D_r S_r), \quad s_e = S_e/S^0, \quad b_e = Y_s s_e, \quad \gamma = Y_S \tag{5}$$

and

$$\rho(s) = \frac{K(S_r s)}{D_r} = \frac{k_0 s}{s^2/k_i + s + k_s}, \quad k_0 = K_0/D_r,$$

$$k_i = K_i/S_r, \quad k_s = K_s/S_r, \quad s^* = \sqrt{k_s k_i} = S^*/S_r,$$

$$\rho^* = \rho(s^*) = k_0/(1 + 2\sqrt{k_s/k_i}), \tag{6}$$

s (or b) is the dimensionless substrate (or biomass) concentration, t is the dimensionless time referred to the reference residence time D_r^{-1}, θ is the dimensionless dilution rate, and p is the dimensionless biomass production rate. The value s^* is the substrate concentration where the growth rate $\rho(s)$ reaches its maximum ρ^* (see Fig. 1) (Andrews, 1968).

In compact control notation the bioreactor system (4) is written as follows:

$$\dot{x} = f(x,u,d), \quad y = x_1 + \tilde{y}, \quad x(0) = x_0,$$

$$x = (s,b)', \quad d = (\kappa', s_e)', \quad \kappa' = (k_0, k_s, k_i, \gamma), \quad u = \theta, \tag{7}$$

where x is the substrate-biomass state, u is the dilution rate control input, y is the biomass measured output with error $\tilde{y}$, and d is the exogenous input with (possibly time-varying) entries κ and s_e.

Due to process design considerations, the control input θ can take values between its lower (u^-) and upper (u^+) limits. Accordingly, we are interested in designing a saturated robust stabilizing