Chemical Engineering Science 60 (2005) 301–313

Chemical Engineering Science

www.elsevier.com/locate/ces

Further work on cake filtration analysis

Renbi Bai[a,*], Chi Tien[b]

[a]Department of Chemical and Biomolecular Engineering, 10 Kent Ridge Crescent, National University of Singapore, Singapore 119260, Singapore
[b]Department of Chemical Engineering and Materials Science, Syracuse University, Syracuse, NY 13244, USA

Received 24 December 2003; received in revised form 11 August 2004; accepted 12 August 2004
Available online 30 September 2004

Abstract

Analysis of cake filtration was made by the numerical solution of the appropriate equations of change based on the multiphase flow theory with the assumption that the cake properties are functions of the particle phase compressive stress, p_s. Unlike earlier studies which assume the relationship between p_s and the pressure of the fluid phase, p_l, to be $\nabla p_s + \nabla p_l = 0$, other possibilities were also considered in view of the recent work of Tien et al. (Chem. Eng. Sci. 56 (2001) 5361).

In addition to investigating the effect of the p_s–p_l relationship, comparisons of predicted filtration performance with experiments made it possible to substantiate earlier findings that the p_s–p_l relationship is system specific. The results of the analysis were also used to test the parameter sensitivity of predictions, namely, values of the parameters of the constitutive relationships (i.e. ε_s vs. p_s and α vs. p_s, where ε_s and α are the cake solidosity and specific cake resistance). This information, in turn, can be used as a bench mark for improving existing and developing new procedures for determining cake solidosity and permeability.

Keywords: Cake filtration; Analysis; p_s–p_l Relationships; Filtration performance; Cake structure

1. Introduction

Analysis of cake filtration has been a subject of study for several decades. The pioneer work of Ruth et al. (1933a,b) and subsequent investigations of Grace (1953a,b), Tiller and Shirato (a summary of their studies can be found in Shirato et al., 1987) led to the development of the conventional cake filtration theory. For further improvement of the conventional theory, more rigorous analyses based on the solution of the volume-averaged equations of continuity have been attempted in recent years (Atsumi and Akiyama, 1975; Wakeman, 1978; Tosun, 1986; Stamatakis and Tien, 1991; Landman et al., 1995; Tien et al., 1997; Burger et al., 2001; Ramarao et al., 2002; Tien and Bai, 2003). The results of these analyses have demonstrated their utility not only in predicting filtration performances but also providing information about cake structure, its evolution and the effects of sedimentation and fine particle retention.

The complexities of cake filtration arise mainly from the compressible behavior of filter cakes. For a given cake, the extent of its compression is determined by the compressive stress to which it is subject with this compressive stress resulting from the cumulative drag forces acting on cake particles. The relationship between the compressive stress, p_s, and the pore liquid pressure, p_l, for the one-dimensional rectilinear case is commonly assumed to be

$$\mathrm{d}p_l + \mathrm{d}p_s = 0. \tag{1}$$

As pointed out by several investigators before (for example, Rietema, 1982), the relationship given by Eq. (1) is just one of several possibilities. This fact was confirmed by the recent work of Tien et al. (2001), who showed that better agreement between cake filtration results and the results of the compression–permeability (C–P) measurements can be established using other types of p_s–p_l relationships instead of that of Eq. (1).

* Corresponding author. Tel.: +65-6874-4532; fax: +65-6779-1936.
E-mail address: chebairb2@nus.edu.sg (R. Bai).

0009-2509/$ - see front matter © 2004 Published by Elsevier Ltd.
doi:10.1016/j.ces.2004.08.008

The purpose of the present study is to make a detailed examination on the effect of the p_s–p_l relationship in the analysis of cake filtration. Three different p_s–p_l relationships considered by Tien et al. (2001) were used for the numerical solutions of the volume-averaged equations and the results obtained were compared. Furthermore, by comparing the numerical results with experiments, appropriate p_s–p_l relationship was identified and corroborations with the earlier work of Tien et al. (2001) were made. The numerical results were also used to assess the validities of some of the assumptions used by investigators previously.

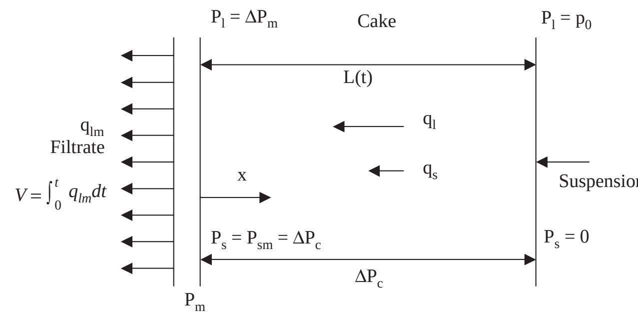

Fig. 1. Schematic representation of cake filtration.

2. Analysis

The volume-averaged equations of continuity of mass and momentum of multiphase systems are

$$\rho_i \frac{\partial \varepsilon_i}{\partial t} = -\rho_i \nabla \cdot (\varepsilon_i \underline{U}_i) + m_i, \tag{2}$$

$$\rho_i \varepsilon_i \frac{\mathrm{D}}{\mathrm{D}t} \underline{U}_i = -\underline{S}_i + \underline{w}_i + \underline{F}_{ji}, \tag{3}$$

where the subscript i stands for $\ell =$ liquid phase and $s =$ particle phase. ε_i is volume fraction of phase i, ρ the density, $\underline{U}$ the mass-average velocity vector, $\underline{S}$ the stress vector and $\underline{w}$ the body force vector. $\underline{F}_{ji}$ is the interaction force vector between phases j and i. And m_i is the net mass flux into phase i.

For one-dimensional cake filtration in rectilinear coordinate as depicted in Fig. 1, Eq. (2) may be written as follows:

$$\frac{\partial \varepsilon}{\partial t} = -\frac{\partial q_l}{\partial x}, \tag{4a}$$

$$\frac{\partial \varepsilon_s}{\partial t} = -\frac{\partial q_s}{\partial x}, \tag{4b}$$

where q_l and q_s are the liquid and particle velocities. ε_ℓ is now written as ε (or cake porosity). x is the distance measured from the medium. With this coordinate system, q_l and q_s are inherently negative.

The generalized Darcy's law is used to relate the relative liquid/solid velocity with the gradient of p_l or

$$\frac{q_l}{\varepsilon} - \frac{q_s}{\varepsilon_s} = -\frac{1}{\varepsilon} \frac{k}{\mu} \frac{\partial p_l}{\partial x}, \tag{5a}$$

which, upon rearrangement, yields

$$q_l = -\varepsilon_s \frac{k}{\mu} \frac{\partial p_l}{\partial x} + (1 - \varepsilon_s) q_{\ell_m}. \tag{5b}$$

Substituting the above expression into Eq. (4a) and noting $\varepsilon = 1 - \varepsilon_s$, one has

$$\frac{\partial \varepsilon_s}{\partial t} = \frac{\partial}{\partial x} \left[-\varepsilon_s \frac{k}{\mu} \frac{\partial p_l}{\partial x} \right] - q_{\ell_m} \frac{\partial \varepsilon_s}{\partial x}, \tag{6}$$

where q_{ℓ_m} is the instantaneous filtration velocity and is given as

$$q_{\ell_m} = \left[-\frac{k}{\mu} \frac{\partial p_l}{\partial x} \right]_{x=0} = \frac{-p_{\ell_m}}{\mu R_m}, \tag{7}$$

where p_{ℓ_m} is the pressure at the cake/medium interface (i.e. $x = 0$) and R_m is the medium resistance.

In Eq. (6), ε_s and k are quantities characteristic of the cake in question and are commonly assumed to be functions of the compressive stress, p_s. As stated before, there is a relationship between p_l and p_s, which can be found from Eq. (3). If one ignores the inertial effect and the presence of the body force, by adding the two equations corresponding to the liquid and particle phase, and noting $\underline{F}_{ij} = -F_{ji}$, one has

$$\underline{S}_\ell + \underline{S}_s = 0. \tag{8}$$

S_i $(i = \ell, s)$, the force vector on phase i results from the stress tensor $\underline{\underline{T}}_i$ of the same phase. $\underline{\underline{T}}_i$ may be written as

$$\underline{\underline{T}}_i = -p_i \underline{\underline{\delta}} + \underline{\underline{\tau}}_i, \tag{9}$$

where p_i is the isotropic pressure of phase i and $\underline{\underline{\delta}}$ the unit tensor. $\underline{\underline{\tau}}_i$ is the shear stress tensor of the phase i.

In deriving the volume-averaged momentum continuity equations, different relationships between S_i and $\underline{\underline{T}}_i$ have been proposed such as (Rietema, 1982)

$$\underline{S}_i = \nabla \cdot \underline{\underline{T}}_i, \quad \nabla \cdot \varepsilon_i \underline{\underline{T}}_i \quad \text{or} \quad \varepsilon_s \nabla \cdot \underline{\underline{T}}_i, \quad i = \ell \text{ or } s. \tag{10a}$$

In addition, for the dispersed phase (i.e. particle phase), the following relationships have also been proposed

$$\underline{S}_s = \varepsilon_s (\nabla \cdot \underline{\underline{T}}_\ell + \nabla \cdot \underline{\underline{T}}_s) \quad \text{or} \quad \varepsilon_s \nabla \cdot \underline{\underline{T}}_\ell + \nabla \cdot \underline{\underline{T}}_s. \tag{10b}$$

Thus by choosing among the various definitions given above, a variety of p_s–p_l relationships between p_s and p_l can be established. Furthermore, for one-dimensional cake filtration, the pressure terms (p_s and p_l) are dominant. Three of the simplest cases considered by Tien et al. (2001) are

$$\mathrm{d}p_l + \mathrm{d}p_s = 0 \quad \text{(Type 1)}, \tag{11a}$$

$$(1 - \varepsilon_s)\mathrm{d}p_l + \mathrm{d}p_s = 0 \quad \text{(Type 2)}, \tag{11b}$$