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Overall map and correlation of dispersion data for flow through granular packed beds

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Abstract

Measured values of the coefficients of transverse and longitudinal dispersion (D_T and D_L, respectively) are reported for liquid flow through granular packed beds. Measurements of D_T were made for $50 < Sc < 1950$ and $300 < Pe_m < 10^5$, working with water at temperatures between 278 and 373 K, and values of D_L were measured for $Sc \cong 750$ and $1 < Pe_m < 45$, working with water at 293 K; nearly two hundred new data points are reported.

The data obtained, together with data from other sources, both for gas and liquid flow, are reported in plots of Pe_T vs Pe_m and Pe_L vs. Pe_m, in order to stress the influence of Sc on dispersion and elucidate the difference between liquid and gas behaviour.

Empirical correlations are presented for the prediction of the dispersion coefficients (D_T and D_L) over the entire range of practical values of Sc and Pe_m, and they are shown to give the dispersion coefficients with very good accuracy.

Keywords: Transverse dispersion; Longitudinal dispersion; Packed beds; Mass transfer; Granular beds; Schmidt number

1. Introduction

The problem of solute dispersion in flow through packed beds has attracted interest from the early days of this century (Slichter, 1905), but it was only since the 1950s that the general topic of hydrodynamic dispersion, or miscible displacement, became the subject of more systematic study. This topic has interested hydrologists, geophysicists, petroleum and chemical engineers (e.g., Levenspiel, 1962), among others, and for some time now it has been treated at length in books on flow through porous media (e.g., Dullien, 1979; Scheidegger, 1974; Bear, 1972).

The quantitative treatment of dispersion is currently based on the use of an equation with the form of Fick's law, with the appropriate dispersion coefficients taking the place of the molecular diffusion coefficient; cross-stream dispersion is accounted for by the transverse dispersion coefficient, D_T, whereas streamwise dispersion is accounted for by the longitudinal dispersion coefficient, D_L.

In the limit of very low fluid velocity, dispersion is determined solely by molecular diffusion, with $D_T = D_L = D_m/\tau$ (τ being the tortuosity factor for diffusion). In the opposite limit of turbulent flow, dispersion becomes purely "fluid mechanical" and it is well known (see Hiby, 1962; Wilhelm, 1962; Bischoff, 1969; Coelho and Guedes de Carvalho, 1988) that upon increasing the velocity of the fluid, the dimensionless groups $Pe_T = ud/D_T$ and $Pe_L = ud/D_L$ tend to the approximate limiting values of $Pe_T(\infty) \cong 12$ and $Pe_L(\infty) \cong 2$, in packed beds of near spherical particles.

The equations

$$\frac{1}{Pe_T} = \frac{1}{\tau}\frac{1}{Pe_m} + \frac{1}{12} \tag{1}$$

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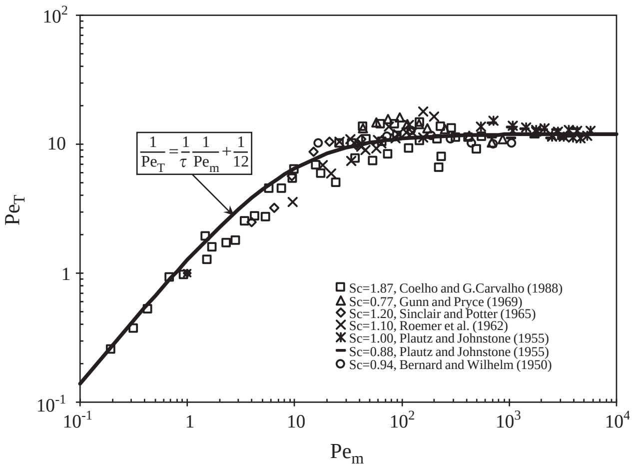

Fig. 1. Data on transverse dispersion in gas flow.

and

$$\frac{1}{Pe_L}=\frac{1}{\tau}\frac{1}{Pe_m}+\frac{1}{2} \tag{2}$$

with $Pe_m = ud/D_m$ are obtained by simply adding the contributions of molecular diffusion and turbulent dispersion and they give the correct asymptotic behaviour (both for very high and very low Pe_m) for both gases and liquids, as reported by several workers (see Gunn, 1969; Wilhelm, 1962). In the intermediate range of Pe_m they are still a reasonable approximation for gases, the wider deviation being observed in the intervals $3 < Pe_m < 300$, for Pe_T and $0.5 < Pe_m < 100$, for Pe_L, as shown in Figs. 1 and 2.

With liquids, deviation from Eqs. (1) and (2) is pronounced over an extended range of values of Pe_m (say, $10 < Pe_m < 10^6$), as may be gathered from Figs. 5 and 11 in the present paper. The difference between "gas behaviour" and "liquid behaviour" has to do with the value of $Sc(=\mu/\rho D_m)$. Indeed, dimensional analysis shows (see Guedes de Carvalho and Delgado, 2000) that Pe_T and Pe_L are functions of Pe_m and Sc, values of Sc being of order 1 for most gas mixtures and of order 10^3, and higher, for most liquids close to room temperature. Quite surprisingly, no data on dispersion seem to have been published until recently, for the range $2 < Sc < 550$. Delgado and Guedes de Carvalho (2001) seem to have been the first to provide data on transverse dispersion in the range $50 < Sc < 500$, but their study was restricted to the range $Pe_m < 1350$, due to limitations of the experimental technique adopted. In the present work the alternative technique of continuous tracer injection (Hiby, 1962) was introduced to extend the study of the influence of Sc on D_T to the range $10^3 < Pe_m < 10^5$.

The influence of Sc on D_L was studied by Guedes de Carvalho and Delgado (2003) in the approximate ranges $50 < Sc < 1950$ and $20 < Pe_m < 10^5$. Plots of Pe_L vs Pe_m (their Figs. 8 and 10) showed that Pe_L seems to become independent of Sc (over the range of Sc studied), as the value of Pe_m is decreased to about 20. But since the value of Pe_L is then still considerably lower than the value predicted by Eq. (2), in the present work we undertook to measure values of D_L in the range $1 < Pe_m < 40$; as reported below, this required an adaptation of the technique followed by Guedes de Carvalho and Delgado (2003).

The vast amount of data obtained by our group, together with the numerous data available from other sources, mostly for air and water at room temperature, provide a very detailed representation of the functions $Pe_T = f_1(Pe_m, Sc)$ and $Pe_L = f_2(Pe_m, Sc)$. In the present paper, we put forward mathematical expressions that represent the data now available with good accuracy and they are a significant improvement over previous correlations.

2. Transverse dispersion

In order to widen the range of Pe_m covered by Delgado and Guedes de Carvalho (2001), in their study of the influence of Sc on D_T, it was decided to explore the technique used by Hiby (1962), based on the interaction between two parallel streams, one of pure water and the other of water containing a small amount of tracer. Fig. 3 sketches the principle on which the experimental method is based and it is important that values of tracer concentration are kept low, to avoid density-induced flow effects. Also, the tracer has to be conserved and the distribution of flow rates at the outlet from the bed must be the same as in the feed, so as to give a uniform flow field across the packing.

2.1. Theory

Taking a radial co-ordinate, r, to measure distance to the axis of symmetry of the cylindrical packed bed and a co-ordinate, z, parallel to the axis, the differential mass balance on the solute reads

$$\frac{D_T}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right)+D_L\frac{\partial^2 C}{\partial z^2}=u\frac{\partial C}{\partial z}, \tag{3}$$

where D_T and D_L are the radial and longitudinal dispersion coefficients, respectively. The contribution of longitudinal dispersion may be neglected since the boundary layer for mass transfer is thin in comparison with the axial distance (L) along which the two concentric streams are allowed to contact. Eq. (3) then reduces to

$$\frac{D_T}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right)=u\frac{\partial C}{\partial z} \tag{4}$$

and for the situation sketched in Fig. 3, Eq. (4) is to be solved with

$$z>0, \qquad r=R, \qquad \frac{\partial C}{\partial r}=0, \tag{5a}$$

$$z=0, \qquad r<R_i, \qquad C=C_0, \tag{5b}$$

$$z=0, \qquad R_i<r<R, \qquad C=0. \tag{5c}$$