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Finite-element modelling of combined free/porous flow regimes: simulation of flow through pleated cartridge filters

V. Nassehi*, N.S. Hanspal, A.N. Waghode, W.R. Ruziwa, R.J. Wakeman

Chemical Engineering Department, Advanced Separation Technologies Group, Loughborough University, Loughborough, Leicestershire, LE11 3TU, UK

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Abstract

Mathematical modelling of creeping incompressible Stokes flow and low-permeability Darcy flow are well established and a number of reliable schemes for the simulation of these regimes are available in the literature. However, modelling of combined Stokes/Darcy regimes, such as those encountered in many types of industrial filters, still presents mathematical and practical challenges. In this paper, we present a finite-element model for the prediction and quantitative analyses of the hydrodynamic behaviour of deadend pleated cartridge filters. Elemental discretisation in this scheme is based on the use of unequal order approximation functions for velocity and pressure fields. We show that this discretisation generates unified stabilisation for both Stokes and Darcy equations and prevents 'numerical locking' whilst preserving the geometrical flexibility of the computational grid. Conducting a number of numerical tests, it is shown that the developed model is capable of yielding theoretically expected and accurate simulations for realistic industrially relevant problems. The model is tested for shear thickening non-Newtonian fluids, which represent fluids used in aeronautical applications and in some process industries. This study is part of a multi-disciplinary project undertaken by various investigators for the design and development of high-performance deadend pleated cartridge filters for aeronautical applications. It has been demonstrated that the developed model presents a cost effective, robust and reliable design tool to enable engineers to appraise the operation of such filters.

Keywords: Pleated cartridge; Hydrodynamics; Deadend filter; Porous media; Filtration; Finite-element method

1. Introduction

Recently, there has been an increasing interest in the design of filter cartridges that have a lower environmental impact through their use and reduced disposal costs. The present study focuses on filtration of hydraulic fluids used in aeronautical applications. Highly viscous non-Newtonian fluids such as ester phosphates are employed to actuate the hydraulic components in a typical aircraft system. To avoid wear and tear of expensive machine devices from specific contaminants, these fluids are filtered continuously during such an application. The disposal of the blinded used cartridges is a matter of serious environmental concern. Therefore, there is a need to design filter elements, which can sustain high pressures and can be safely disposed after use. In conventional filters the cartridge is clamped between metallic flanges, which results in disposal problems. The new concept aims at the replacement of these metallic parts by novel composite materials so that the elements can directly be incinerated after use. The performance of filter media in these cartridges should be reliably predicted to enable the engineers involved in the design of cartridge filters for the aircraft industry to incorporate such novelties. The present work focuses on the development of a robust and cost effective computer model for this purpose. The flow regimes in a deadend cartridge filter assembly consist of creeping free flow and porous flow regions. These regimes are represented by the Stokes and Darcy equations, respectively. Therefore, there is a need to accurately represent both

* Corresponding author. Tel.: +44 1509 222522; fax: +44 1509 223923.
E-mail address: V.Nassehi@lboro.ac.uk (V. Nassehi).

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of these regimes in a single model in order to predict the velocity field variations and pressure distribution in the filter.

Studies in combined free and porous flow systems have received considerable attention in the last two decades. However the numerical models developed for the problem are based on ad hoc approaches and cannot be universally applied. The main challenge is to construct a sound mathematical formulation for the coupled free and porous flow dynamics (Layton et al., 1993). This is because that the differential operators appearing in the Stokes and Darcy equations are essentially incompatible. The selection of appropriate approximating function spaces for the numerical solution of these equations which guarantee unified stability of both the models is not straightforward (Burman and Hansbo, 2002). In order to circumvent this problem, most of the solution algorithms in the literature are based on two approaches. The first approach employs the Brinkman's equation to describe the flow on a free/porous interface (Brinkman, 1947). The Brinkman's equation includes the Laplacian of the velocity field and hence is of the same order as the Stokes equation. Therefore its use across a free/porous interface ensures continuity of velocity and pressure fields. However, the Brinkman's equation is only applicable to media of high permeability (Kim and Russell, 1985). In addition, the Brinkman's equation is constructed using a parameter called "effective viscosity'' which, in general, cannot be experimentally measured. The second approach is based on the imposition of a slip-wall boundary condition at the free/porous interface as suggested by Beavers and Joseph (1967). Using this approach, the interfacial boundary condition is modified to match the free and porous flow conditions by an auxiliary relationship. This is an empirical approach deduced from a simple one-dimensional situation and its extension to multi-dimensional cases is not well understood. However, in practice many efforts have been made to obtain such extensions. For example, Saffman (1971) extended the Beavers and Joseph method by applying it to a non-homogenous porous medium via a statistical approach. Later, Jäger and Mikelć (2000) provided some mathematical justification for Saffman's form of the Beavers and Joseph condition. In spite of all such work, the use of Beavers and Joseph approach poses an unresolved problem because it depends on the evaluation of a slip coefficient at the free/porous interface. The value of the slip coefficient depends on many physical parameters such as the geometrical features of the interface, making it nearly impossible to measure experimentally.

To overcome such complications, recently Burman and Hansbo (2002) proposed a unified approach for the solution of the coupled Stokes/Darcy problem using an edge stabilisation technique in conjunction with the use of mixed P_1/P_0 finite elements. The edge stabilisation, as named by these authors, involves the splitting of the viscous stress term into its normal and tangential components at the free/porous interface. The normal stress component is then equated to the pressure differential across the free/porous interface. The concepts illustrated are mathematically complicated for their suitability for practical situations where the interface may have complex geometrical shapes. Similarly, Masud and Hughes (2002) have examined many specialised finite-element schemes for the stabilised solution of the Darcy equation. Though both these schemes stand firm on grounds of stability and convergence, there is no evidence of their applicability to flows in highly complex and irregular geometries such as aeronautical pleated cartridge filters.

Another strategy for this problem, employed mainly by engineers, is to make simplifications in the geometry or flow regime characteristics in free/porous domains. There are many examples of such an approach in the literature of which the following two are typical. Chen et al. (1995) developed a numerical model to optimise the design of pleated filter panels. The pleats are taken to be simple rectangles. In addition, uniform velocity profiles are assumed in the upstream pleat channel and sinusoidal in downstream channel. It is further assumed that the fluid penetrates in the porous medium only at the top and bottom regions of the pleat, any penetration along the length of the pleat is negligible. The free flow is modelled by Stokes equations while the porous flow is modelled by Darcy–Lapwood–Brinkman equation to compensate for the simplified velocity profiles. More recently, Oxarango et al. (2004) developed a one-dimensional model to determine laminar flow of a fluid in porous channels with wall suction or injection. They extended their approach to pleated filters by a two-dimensional model using a unit element of the pleated filter with periodic boundary conditions. The pleats considered in their work are again of rectangular geometry and the intrusion of the fluid in porous media is assumed to be unidirectional. However, such simplifications reduce the applicability of these models to realistic industrial situations where the flow geometry is usually complex and flow regime is multi-dimensional and highly non-uniform.

In this paper, we present a Galerkin finite-element scheme based on unequal order approximating functions for the velocity and pressure fields, which can model the coupled free and porous flow regimes efficiently. This scheme preserves the numerical stability of the coupled flow problem and avoids any mathematical complications. Numerical tests provide evidence for the theoretical consistency and accuracy of the predictions of the present model for geometrically complex domains.

2. Mathematical statement of the combined Stokes/Darcy flow regimes

Consider a flow model consisting of the following equations:

$$\boldsymbol{A}(\vec{u}) + \vec{\nabla} p = \vec{f}, \tag{1}$$

$$\vec{\nabla} \cdot \vec{u} = 0 \tag{2}$$