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Intrinsic instability of flame–acoustic coupling

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ABSTRACT

This paper shows that a flame can be an intrinsically unstable acoustic element. The finding is clarified in the framework of an acoustic network model, where the flame is described by an acoustic scattering matrix. The instability of the flame acoustic coupling is shown to become dominating in the limit of no acoustic reflections. This is in contrast to classical standing-wave thermoacoustic modes, which originate from the positive feedback loop between system acoustics and the flame. These findings imply that the effectiveness of passive thermoacoustic damping devices is limited by the intrinsic stability properties of the flame.

1. Introduction

The development of combustion devices is often limited by the occurrence of so called thermoacoustic instabilities. Such instabilities are induced by a strong coupling between the acoustic field and the flames in a combustor, and may generate unacceptable noise levels or even lead to structural damage [1]. Although major steps in the understanding and modelling of thermo-acoustic instabilities have been made in the last decades there are still numerous technological and scientific challenges [2–5].

In general, thermoacoustic instabilities are the result of a complex interplay between various physical processes with a wide range of time and length scales [6]. Although the recent advances in computing power and computational fluid dynamics have made it possible to simulate complete combustors up to the finest scales, this is still a time-consuming task.

Alternatively, it is possible to model a complete system as a combination of various physical sub-elements. It is immediately clear that within such a framework, one may distinguish between three different system levels in the complete model hierarchy, namely (i) total system as complete network of sub-elements, (ii) single element level, e.g. the relevant input–output behavior of an element, (iii) internal element level, e.g. the detailed mechanisms which constitute the relevant response at (ii).

For thermoacoustic systems modelled in the framework sketched above it is then natural that one may study the behavior of the (sub)-systems at any of the levels (i), (ii) or (iii). For example, on the deepest level one can study the complicated interaction between local (turbulent) flow dynamics, chemical kinetics, and heat release [7]. On the element level one may be interested in the acoustical response of an unflanged open end [8], or the acoustical response of a flame [2]. Finally, there is a wealth of literature on the analysis of thermoacoustic systems on a complete system level, see for example the various linear and nonlinear studies in [3,9–12].

Nevertheless, in any complete thermoacoustic study, the flame plays an essential role, which intrinsic behavior is also the focus of the current contribution. The essence of the flame–acoustic coupling has been known since Rayleigh [13], who first clarified how a fluctuating heat release can act as an acoustic source when the pressure and heat release fluctuations are in phase. The acoustic feedback loop is closed when the heat release fluctuation is the result of a fluctuation of acoustic variables somewhere in the system. Naturally, the excitation mechanism may vary depending on the type of burner and flames used [2]. Notwithstanding other possible mechanisms, the flame sub-element typically takes the form of a transfer function between the acoustic velocity upstream, and the heat release from the flame. Here one has to note that such a transfer function is in fact a linear approximation to the flame dynamics, and as such is only suitable for linear system models. The flame transfer function has been the subject of numerous research efforts, which brought to light many of the physical interactions and parameters responsible for typical flame transfer functions, see for example [14–19].

In general, the complete thermo-acoustic model can then be built following two possible methods. In terms of the acoustic Helmholtz equation, the flame transfer function is used in the

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inhomogeneous source term, which due to the velocity becomes directly coupled to the field, see for example [20]. The final equation, with appropriate boundary conditions, can then be solved in a Finite Element (FEM) framework. Alternatively, in order to build a one-dimensional acoustic network model, the flame transfer function is combined with the Rankine Hugionot jump conditions, [21], to form a flame transfer matrix, with pressure and velocity up- and downstream of the flame as in- and outputs.

In contrast to the extensive research on the flame transfer function, the properties of the complete description of the flame–acoustic coupling, e.g. the flame transfer matrix, are hardly investigated. One notable exception is [22], where the instability potential of a burner in a complete system was quantified by calculating the maximum and minimum possible amplification from a given flame scattering matrix. The results indicate that the instability potential depends strongly on the used flame transfer function inside the matrix.

In a similar spirit as in [23,22], the main aim of the current contribution is to clarify the fundamental properties of the flame element. However, instead of focusing on the potential acoustic amplification of a burner, we focus on the stability properties of the burner/flame element itself. Thus, our investigation is mainly on the second level (ii) in the model hierarchy.

It is shown that the flame–acoustic coupling can have an intrinsically unstable behavior. It is illustrated that the (in)stability of the flame element dominates the complete system behavior in case of large acoustic losses. The fact that the flame may be unstable even in the absence of acoustic feedback is completely unexpected based on the classical presumption that the flame just amplifies the acoustic modes of the combustor. The disclosure of the flame intrinsic instability is the main result of this study.

It should be noted that the main theory is based on, but not limited to, a network modelling framework. The basic results presented here are indeed verified by the authors in a Finite Element Method (FEM) calculation of the inhomogeneous Helmholtz equation.

It is perhaps important to mention that the found intrinsic instability seems of a completely different nature than the extensively studied hydrodynamic and diffusive-thermal instabilities reported in for example [24].

A companion paper [25] deals with the same problem as presented here and provides complementary insight to the physics of the intrinsic flame instability. Their work was done simultaneously and independently from the present study.

The remainder of the paper is organized as follows. Section 2 contains a short introduction and overview of standard acoustic network modelling. Then, in Section 3 the stability of the flame scattering matrix is investigated based on two different flame transfer function models. Section 4 clarifies how the results relate to classical thermoacoustic modes. Finally, Section 5 provides the discussion and conclusions.

2. Acoustic network modelling and flame descriptions

Acoustic network modelling is a widespread standard tool to asses thermoacoustic instabilities. For the sake of completeness the next section gives a very short overview on acoustic network modelling. It also serves to clarify the naming convention used in this paper with respect to the various matrices. Any reader already familiar with the topic may skip the first subsection and continue with Section 2.2.

2.1. Network modelling

In order to apply one-dimensional acoustic network modelling, the frequency range of interest should be smaller than the cut-on frequency of the first transversal mode [26]. In case one then assumes a time dependence in the form $e^{i\omega t}$ with (complex) frequency $\omega = \omega_r + i\omega_i$ it is possible to represent nearly any acoustical element using a set of linear relations. There are different choices possible for the variables used to express the element transfer functions. Here, let us focus on the descriptions involving the Riemann invariants f, g, which can be written in terms of the complex acoustic pressure p' and velocity u' amplitudes using $f = \frac{1}{2}(\frac{p'}{\rho c} + u')$ and $g = \frac{1}{2}(\frac{p'}{\rho c} - u')$, where ρ and c are respectively the densities and speed of sound in the fluid [27].

When the left- and right-travelling waves at one (physical) side of the element are written as function of the ones at the other side, the transfer matrix **T** is obtained:

$$\begin{bmatrix} f_2 \\ g_2 \end{bmatrix} = \begin{bmatrix} T_{11}(\omega) & T_{12}(\omega) \\ T_{21}(\omega) & T_{22}(\omega) \end{bmatrix} \begin{bmatrix} f_1 \\ g_1 \end{bmatrix}. \quad (1)$$

One can arrive at a particularly insightful description if the equations of **T** are rearranged such that the ingoing waves (f_1, g_2) appear as inputs to the matrix, and the outgoing waves (g_1, f_2) as outputs. In that case, the so called scattering matrix **S** can be defined:

$$\begin{bmatrix} g_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} S_{11}(\omega) & S_{12}(\omega) \\ S_{21}(\omega) & S_{22}(\omega) \end{bmatrix} \begin{bmatrix} f_1 \\ g_2 \end{bmatrix}. \quad (2)$$

This description is convenient, since the diagonal elements S_{11} and S_{22} represent the reflection coefficients as seen from the up- and downstream sides of the element, whereas S_{21} and S_{12} are the transmission coefficients to the up- and downstream sides. Note that f, g, p and u are in general the complex amplitudes of the acoustic waves.

A complete system model can be obtained if one combines the individual transfer matrices of the separate elements in a system matrix, where the corresponding variables of adjacent elements are matched. For a single branch system with M elements one then arrives at a system with $2M$ equations with $2M + 2$ unknowns. The final two equations are obtained from the boundary conditions on each side of the network model branch. These take the form of end reflections R in case of f, g variables.

Let us now consider a simple example of the abstract model, in terms of a transfer matrix **T**, shown in Fig. 1. For this case, the total set of system equations is given by,

$$\begin{bmatrix} 1 & -R_1 & 0 & 0 \\ 0 & 0 & -R_2 & 1 \\ T_{11} & T_{12} & -1 & 0 \\ T_{21} & T_{22} & 0 & -1 \end{bmatrix} \begin{bmatrix} f_1 \\ g_1 \\ f_2 \\ g_2 \end{bmatrix} = 0, \quad (3)$$

where we denote the left hand side system matrix as $X(\omega)$.

Finally, the eigenfrequencies and modes are the nontrivial solutions of (3), given by

$$|det(X(\omega)| = 0. \quad (4)$$

In case of the matrix (3), one can easily verify that this leads to the following equation,

$$T_{22} - R_2T_{12} + R_1T_{21} - R_1R_2T_{11} = 0, \quad (5)$$

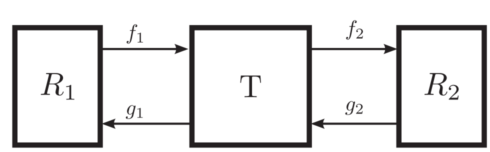

Fig. 1. A simple system in terms of the transfer matrix **T**.