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DNS of Intrinsic ThermoAcoustic modes in laminar premixed flames

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Recent studies [Hoeijmakers et al. 2014, Emmert et al. 2015] suggest that thermoacoustic modes can appear in combustors with anechoic terminations, which have no acoustic eigenmodes. These modes, called here Intrinsic ThermoAcoustic (ITA), can be predicted with simple theoretical arguments, but have been ignored for a long time. They are reproduced in this paper using Direct Numerical Simulation (DNS) of a laminar premixed Bunsen type flame. DNS results and theory are compared showing very good agreement in terms of both frequency and mode structure. DNS confirms that the frequency of ITA modes does not depend on any acoustic characteristic of the burner. Based on a numerical evaluation of the Flame Transfer Function, stability limits of ITA modes predicted by theory are also recovered in the DNS with reasonable accuracy. Finally, DNS is used to analyze the mechanisms of ITA modes.

1. Introduction

Thermoacoustic instabilities have been a topic of strong interest in the aerospace industry as well as many other engineering applications for the past decades. Rayleigh [1] first described the underlying mechanism of thermoacoustic instabilities as a constructive build up of acoustic energy by the product of acoustic pressure and unsteady heat released by the flame. Such accumulation is usually found in systems that can store acoustic energy, thereby involving acoustic eigen modes. Resonance has become a mainstay of combustion instability analysis, leading to the idea that there is no such thing as a thermo-acoustically unstable flame in the absence of acoustic modes in the combustor, and that there has to be some sort of acoustic coupling with the surrounding system for an instability to develop.

However, recent theoretical studies [2,3] and experiments [4] suggest that, even in an anechoic environment, thermoacoustic instabilities may exist. These modes are called here Intrinsic Thermo-Acoustic (ITA) modes because they do not require any acoustic feedback from the boundaries of the burner to exist[1]. They correspond to a feedback mechanism inherent to the flame and its anchoring station, completely independent of the acoustic behavior of the surrounding systems (combustion chamber, injectors, nozzle, compressors, turbines, *etc.*). Even though ITA modes have been evidenced only recently by studying anechoic combustors, they may play a major role in most combustors where they can interact with other feedback mechanisms encountered in standard thermoacoustic modes.

ITA modes have been predicted theoretically and observed experimentally [2–4]. Analyzing them using Direct Numerical Simulation (DNS) is an obvious next step [8]. This is the subject of this paper. In Section 2, the theoretical derivation of ITA modes is recalled, based on the work of Hoeijmakers et al. [2] and Emmert et al. [3], and further emphasis is given on the key parameters controlling this instability. Section 3 presents the numerical strategy to capture intrinsic thermoacoustic instabilities, and details are given on the laminar premixed flame setup used in this study. Since theory suggests that ITA modes are controlled by the cross-section ratio S_2/S_1 between chamber and injection duct (Fig. 1), the Flame Transfer Function (FTF) is computed for four cross-section ratios ranging from 1.5 to 6, and used in the theoretical model for stability and frequency predictions in Section 4. The corresponding DNS are performed in Section 5 and compared to theory in terms of stability and frequency. Good agreement is found with theory, but practical limitations for real configurations are also discussed. Section 6 focuses on the particular case $S_2/S_1 = 2$, which is found to be unstable and used for comparison with theory in terms of frequency and mode structure. The acoustic properties of the burner play no role on ITA modes. This is confirmed by performing the simulation of the same flame in two different burner configurations where the lengths of the injection duct and combustion chamber are changed. Finally, in Section 7, the results from DNS are used to capture and evidence the underlying physical phenomena responsible for intrinsic thermoacoustic instabilities.

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[1] The *intrinsic* terminology used here does not refer to flame front instabilities leading to cellular flames such as the Darrieus-Landau instability or thermodiffusive effects [5,6], which do not require acoustics to develop. It is neither related to parametric instabilities [7] which apply to planar flames.

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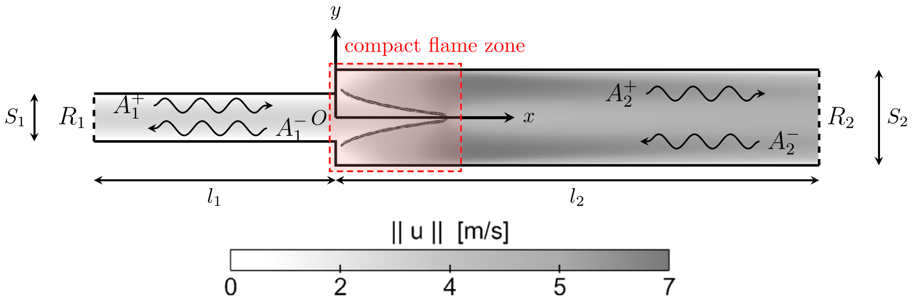

Fig. 1. Configuration of the 2D laminar premixed flame, colored by velocity magnitude. The steady flame position is indicated by a black iso-contour of heat release rate. Downstream and upstream propagating acoustic waves A_j^+ and A_j^- are also represented in the inlet duct ($i = 1$) and the combustion chamber ($i = 2$).

2. Theoretical background

The generic setup studied here corresponds to the classical problem of a laminar premixed flame stabilized at a sudden expansion plane separating the injection duct (length l_1, section S_1) and the combustion chamber (length l_2, section S_2). The configuration is two-dimensional (dihedral flame). The acoustic properties of the inlet and outlet boundaries are represented by reflection coefficients R_j, where the index $j \in \{1, 2\}$ respectively refers to the inlet and outlet ducts.

The linearized Euler equations lead to a simple 1D acoustic wave equation in each duct. The solutions of this equation are forward and backward propagating planar waves A^+ and A^- (see Fig. 1) that couple the acoustic pressure p' and velocity u' fields as follows (assuming harmonic fluctuations):

$$p'_j(x,t) = \Re(\hat{p}_j(x)e^{-i\omega t}) \quad \text{with} \quad \hat{p}_j(x) = A_j^+ e^{ik_j x} + A_j^- e^{-ik_j x} \tag{1}$$

$$u'_j(x,t) = \Re(\hat{u}_j(x)e^{-i\omega t}) \quad \text{with}$$
$$\hat{u}_j(x) = \frac{1}{\rho_j c_j}\left[A_j^+ e^{ik_j x} - A_j^- e^{-ik_j x}\right] \tag{2}$$

where ω is the considered angular frequency, $k_j = \omega/c_j$ the associated wave number, and ρ_j and c_j the density and sound speed that change between cold and hot gases. The *prime* ($'$) and *hat* (^) symbols respectively denote temporal harmonic fluctuations and the associated complex amplitude at the corresponding angular frequency[2]. In the wave formulations of Eqs. (1) and (2) the Mach number is assumed to be zero so that acoustics is not affected by the mean flow. The amplitudes of these planar waves in the injection and combustion ducts are coupled via jump relations to account for the cross-section area change and the combustion source term at $x = 0$, using the assumption that the flame is compact with regards to the acoustic wave lengths. These are known as the acoustical Rankine-Hugoniot jump relations, expressed here in the limit of zero Mach number [9–13]:

$$p'_2(x = 0^+, t) - p'_1(x = 0^-, t) \ = \ 0 \tag{3}$$

$$S_2 u'_2(x = 0^+, t) - S_1 u'_1(x = 0^-, t) \ = \ \frac{\gamma - 1}{\gamma p_0}\dot{\Omega}'_T(t) \tag{4}$$

where γ is the specific heat capacity ratio, p_0 is the reference pressure, assumed to be constant across the flame, and $\dot{\Omega}'_T$ is the fluctuating component of heat release rate integrated over the flame domain. In the velocity jump relation (Eq. (4)), the acoustic emission is due to dilatation induced by the unsteady reaction rate. This relation is expressed in terms of volumetric flow rate, as effects of mean flow on the acoustics are ignored[3] [10,13].

To close the set of Eqs. (3) and (4), $\dot{\Omega}'_T$ must be expressed as a function of the acoustic field. The simplest model for the unsteady reaction rate is to link $\dot{\Omega}'_T$ with the upstream acoustic velocity u'_1 [14]. This model combines a time delay τ and a dimensionless interaction index n:

$$\dot{\Omega}'_T(t) = \frac{\rho_1 c_1^2}{\gamma - 1} S_1 n u'_1(t - \tau) \quad \text{or} \quad \hat{\Omega}_T = \frac{\rho_1 c_1^2}{\gamma - 1} S_1 n \hat{u}_1 e^{i\omega\tau} \tag{5}$$

The term of *velocity sensitive* has been coined for the description of flames whose behavior matches Eq. (5). Experimental data suggest that the heat release of non-planar premixed laminar or turbulent flames is indeed mostly driven by velocity fluctuations rather than pressure fluctuations [15,16]. The parameters n and τ contain all the convective mechanisms controlling the flame response (*e.g.* vortex formation caused by the acoustic velocity surge, vortex convection by the mean flow followed by vortex breakdown and combustion in turbulent flows [17]). Thus, the model of Crocco (Eq. (5)) links the mechanisms in the “convective world” to those in the “acoustic world”, u'_1 being supposed to contain only acoustical components [14]. One may note the peculiar property of this model as it seems to be acoustically non causal[4]: the input quantity being the reference velocity fluctuations in the inlet duct $u'_1 = (A_1^+ - A_1^-)/(\rho_1 c_1)$, it is the difference of upstream and downstream propagating waves. At first sight it seems implausible that an upstream oriented wave A_1^-, propagating away from the flame front, can affect the combustion process. This *causality* concern has been addressed by [8] who showed that Crocco’s model holds for a laminar premixed flame in this configuration, proving that this flame is indeed velocity sensitive. In fact, A_1^- in the inlet duct is the trace of an acoustic wave, generated by the flame or coming from the outlet of the combustor, propagating upstream and that produced a convective wave when passing through the backward step, leading to mode conversion as discussed more thoroughly in Section 7.

Using Crocco’s model (Eq. (5)) and the wave decomposition (Eqs. (1) and (2)), the acoustic jump relations (Eqs. (3) and (4)) become:

$$A_2^+ + A_2^- \ = A_1^+ + A_1^- \tag{6}$$

$$A_2^+ - A_2^- \ = \Gamma(A_1^+ - A_1^-)(1 + n e^{i\omega\tau}) \tag{7}$$

where $\Gamma = (S_1 \rho_2 c_2)/(S_2 \rho_1 c_1)$ is a coupling parameter that reduces to $\Gamma = S_1\sqrt{T_1}/(S_2\sqrt{T_2})$ for isobaric flames using the perfect gas law [12,18,19], T_1 and T_2 being respectively the fresh and burnt gases temperatures.

Eqs. (6) and (7) are the basic elements of all network models for thermoacoustics. ITA modes can be constructed by considering the specific case where both terminations are anechoic: $R_1 = R_2 = 0$. In this case, the amplitudes of the waves propagating towards the flame

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[2] More generally, for any causal temporal signal $g'(t)$, $\hat{g}(\omega)$ would denote the corresponding Laplace transform, defined for any complex frequency $\omega = \omega_r + i\omega_i$.

[3] To take into account mean flow effects (low Mach number regime), a different formulation using mass flow rates should be used instead of volume flow rates in the velocity jump relation.

[4] From an input-output point of view, the temporal notion of causality is respected in Crocco’s model since the input is the reference velocity taken at a previous time $u'_1(t - \tau)$. Only when the acoustic decomposition of the reference velocity is performed does the causality paradox appear.

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