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The acoustic–parametric instability for hydrogen–air mixtures

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ABSTRACT

Acoustic–parametric instabilities are a significant acceleration and self turbulization mechanism which may increase noticeably the propagation velocity of flames. Therefore, the *acoustic–parametric* instabilities for H_2–air mixtures at normal conditions have been investigated. The simplified analytical model proposed by Bychkov as well as the numerical solutions of the Searby and Rochwerger formulation were taken into account. The growth rate of the instabilities and the influence of different fuel concentrations and sound frequencies on the existence of spontaneous transition from the *acoustic* to the *parametric* instability were analyzed. The existence of a wavenumber range in which flames will be unstable for all intensities of sonic perturbations with adequate frequencies was postulated as a consequence of analytic investigation. This constitutes a significant flame acceleration mechanism with major impact on stability and flame development phenomena.

1. Introduction

The interaction of a flame front and an acoustic wave is a process of significant importance in different industrial devices such as turbine combustors or engines and constitutes a challenging problem in combustion science and nonlinear physics [1,2].

The interaction between pressure waves and the flame surface of the flame is a feed-back process in which the wave intensity and the heat released by the flame influence each other. Markstein [3] concluded that coupling between both phenomena is made possible by the variation of the total flame surface caused by the pressure changes. The periodic velocity field created by the waves produces oscillations of the amplitude of the cellular structures of the flames. This variation of the surface alters, in turn, the total amount of fuel consumed and the heat released by the flame which is proportional to the flame area.

Two different instabilities due to flame–pressure waves interaction have been identified [4]. In the *acoustic* instability (see [5]) the cellular structures of the flame front oscillate with the frequency of the acoustic periodic field. Two effects tend to muffle it. For large wavenumbers, the instability is damped by diffusive processes. For small wavenumbers, it is absorbed by the effect of gravity. The *acoustic* instability corresponds, for zero amplitude of the excitation velocity, to the *Darrieus–Landau* planar instability. For increased values of the periodic velocity [5], the *acoustic* instability has the notable property of being able to stabilize the *Darrieus–Landau* instability.

The *acoustic* instability may develop, for enhanced alternating velocity, from the *parametric* one. Under the latter, the growth rate is generally superior to that in the *acoustic* case. The cellular structures of the flame oscillate with a frequency half of the *acoustic*, as the characteristic signal of the Kapitsa *parametrically* dumped pendulum, a fact that was recognized by Markstein who thus named the instability.

From the point of view of the severity of an explosion, gaseous mixtures can be classified in two groups [7]. If the two instabilities co-exist for some ranges of the amplitude of the acoustic perturbation, a planar flame front is never stable and the *acoustic* instability transforms spontaneously into the *parametric* one. If they do not co-exist for any range of acoustic velocities, the *acoustic* instability tends to suppress the Darrieus–Landau instability, the *parametric* instability regime is never reached and planar flame fronts are stable as long as the alternating velocity field exists.

These two different propagation regimes have been confirmed by the observations of Searby [8] and Aldredge and Killingsworth [9] who performed experiments with downward propagating flames inside a cylindrical and an annular burner respectively. It was found that the flame propagation was divided into four separate stages: Promptly after ignition, the flame surface quickly becomes wrinkled due to the *Darrieus–Landau* instability. Then, as the flame propagates further, sound waves start to be generated. Due to the fundamental *acoustic* instability, these acoustic waves cause an attenuation of flame wrinkles. The *Darrieus–Landau* instability is suppressed and the flame becomes planar. Depending on whether the *parametric* and the *acoustic* instability coexist or not, the secondary *parametric* instability may develop producing significant flame acceleration and the appearance of large organized

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Nomenclature

c_p	heat capacity at constant pressure
F	flame front function
g_a	acceleration of gravity
k	wavenumber
L	laminar flame thickness
Le	Lewis number
Ma	Markstein number
Pr	Prandtl number
T_b	temperature combustion products
T_u	temperature reactants
U_a	velocity of the acoustic field
U_L	laminar flame velocity
ϑ	dimensionless temperature
θ	expansion ratio
λ	thermal conductivity
ρ	density
σ	growth rate
χ	thermal diffusivity
ω	frequency acoustic field

pulsating cellular structures. If they do, the final stage of development is characterized by those coherent cellular structures transforming into incoherent flame surfaces fluctuations.

Therefore, gaseous mixtures prone to the *parametric* instability may suffer, in closed chambers, a very significant acceleration of the flame propagation velocity. Especially for lean mixtures, this increase of the combustion rate will be very considerable.

Due to the wide flammability limits of hydrogen–air mixtures [10], which represent a high potential for flame acceleration, an evaluation of the stability characteristics of this gas with respect to the *acoustic* and *parametric* instability is necessary. To this respect, the emphasis of the previous investigations that had experimentally analyzed the *acoustic* and *parametric* instability has been dedicated to hydrocarbons, i.e., propane [4] and methane [9,11], and only recently data become available for hydrogen [12]. From the theoretical point of view, no monographic study has yet been devoted to this problem. The wide flammability limits of hydrogen–air mixtures result in a range of concentrations of practical interest which cover conditions in which $Le < 1$ and $Ma < 0$ (lean mixtures) as well as $Le > 1$ and $Ma > 0$ (rich mixtures). Thus, the scope of the range of these dimensionless numbers for hydrogen flames exceeds the extent of the typical one obtained with hydrocarbons. The significance of this research is even increased by the findings obtained which confirm the unstable character of hydrogen lean mixtures with respect of the *acoustic* and *parametric* instability, a phenomenon that can drive to a violent and spontaneous turbulization of the flame front. This was already experienced by Searby [4], for other gases, and was attributed to the flame-*acoustic parametric* instability. In our investigation, we explicitly restrict ourselves to the analysis of the effects of the acoustic field on a planar flame. In a classical way, we disregard the effect of the flame on the acoustic field, the so called two ways coupling (acoustic-flame and flame-acoustics) already analyzed in [13–15], and the shape of the confinement [15]. We concentrate consequently on the physical properties of a flame suffering an imposed, and unaltered, cyclic velocity.

In view of these considerations, we analyze the *acoustic* and *parametric* instability on hydrogen mixtures at normal conditions, studying the suitability of the methods of analysis, the superposition of both instabilities and its dependencies on different variables. It will be shown, that based on our theoretical analysis it is possible to postulate the existence of a region of instability virtually happening for noise with amplitude tending to zero for gases with $Le < 1$. Assuming that this hypothesis is correct, it is thus shown that a perturbation of suitable frequency acting on a hydrogen–air mixture of the convenient concentration, even with negligible amplitude, can de-stabilize a flame otherwise stable.

To show these findings this paper is organized as follows: the basic equations, methodologies and numerical procedures used in the analysis are described in Section 2; in Section 3 the application of the methodology for hydrogen is presented and in Section 4 extensive conclusions are given. The paper concludes with a brief summary.

2. Analysis

In the present study, two methodologies are applied to study the *acoustic–parametric* instability. The suitability and accuracy of the simplified and convenient solutions obtained by Bychkov [16] can be assessed comparing with numerical solutions of the Searby and Rochwerger formulation.

2.1. Searby and Rochwerger analysis

The mathematical treatment of the *acoustic* and *parametric* instabilities is based on the work of Pelce and Clavin [6], who obtained an equation for a perturbed flame front in a gravitational field under the assumptions of high activation energy and large scale wrinkling. Based on those results, Searby and Rochwerger [7] managed to derive equations for the growth rate of the *acoustic* and *parametric* instability and were able to calculate the stability limits for both cases numerically.

For the sake of simplicity and contrary to the formulation contained in [17] due to Bychkov it was explicitly decided to ignore the effects created by the turbulence standing to the classic formulation based on the works of Searby and Clavin [18]. In such formulation we may consider an infinitely thin flame propagating in vertical direction. The flame front is represented by the function $F(x,t) = 0$ in a reference frame moving with the flame front. The small perturbations could be written in the form $F(x,t) = F(t)\exp(ikx)$. Considering the linear stability problem, the second order differential equation (1), [19] describes the evolution of perturbations of flame surface of small amplitude for periodic monochromatic velocity fluctuations normal to the flame front,

$$A\frac{d^2F}{dt^2} + U_L kB\frac{dF}{dt} + kg_a C_1 F - k\omega U_a \cos(\omega t) C_1 F + U_L^2 k^2 C_2 F = 0, \quad (1)$$

for a gas of arbitrary characteristics. In this equation, A, B, C_1, C_2, are dimensionless coefficients which take the form,

$$A = \left(1 + \frac{\theta - 1}{\theta + 1} kL\left(Ma - \frac{\theta}{\theta - 1}J\right)\right), \quad B = \left(\frac{2\theta}{\theta + 1}(1 + \theta kL(Ma - J))\right), \quad (2)$$

$$C_1 = \left(\frac{\theta - 1}{\theta + 1}\left(1 - kL\left(Ma - \frac{J\theta}{\theta - 1}\right)\right)\right), \quad (3)$$

$$C_2 = \left(\theta\frac{\theta - 1}{\theta + 1}\left(-1 + \frac{kL}{\theta - 1}((3\theta - 1)Ma - 2J\theta + 2Prh_b(\theta - 1) - I(2Pr - 1))\right)\right). \quad (4)$$