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# Mathematical modeling for immersion chilling and freezing of foods. Part II: Model solution

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**Abstract**

A mathematical model to predict the heat and mass transfer during the immersion chilling and freezing of foods was solved using a finite difference method. The control–volume approach with a logarithmic grid was used. Equations and stability criteria were obtained for 1-, 2-, and 3-dimensional regular geometries. Sensitivity analysis showed that the lower temperature or solute concentration of the immersion solution, the faster freezing rate and the slower solute uptake. It was also observed that the higher heat transfer coefficient or the lower diffusion coefficient, the lower solute average concentration in the solid. The logarithmic grid helped conveniently in the representation of the changes that occur near the surface. This work contributes with a simple solution of the model for predicting heat and mass transfer phenomena during immersion chilling and freezing of foods of regular geometries.


*Keywords:* Immersion freezing; Heat transfer; Mass transfer; Numerical solution

## 1. Introduction

Immersion chilling and freezing (ICF) consists of direct soaking of foods in aqueous fluids (e.g. solutions of NaCl, $CaCl_2$ or sucrose) maintained at low temperature (e.g. from −10 to −40 °C). ICF has recognized advantages, it is one of the fastest chilling and freezing techniques, and it is associated to lower costs and to higher quality of the final product. However, the main disadvantage that reduces ICF use is the uncontrolled solute uptake from the refrigerated solution into the product (Lucas & Raoult-Wack, 1998). Mathematical models may help to have a better understanding of the transport phenomena associated with ICF and to control or optimize the variables of the ICF process. The mathematical formulation represents complex phenomena of heat and mass transfer with phase change where the food properties strongly depend on temperature and composition. Zorrilla and Rubiolo (2004) developed a mathematical model based on transport equations for porous media to represent the transport phenomena during ICF process for multidimensional geometries.

The objectives of this work were to solve the mathematical model described in Zorrilla and Rubiolo (2004) and to perform sensitivity analysis considering the main heat and mass transfer parameters.

## 2. Numerical solution

Zorrilla and Rubiolo (2004) developed a model for freezing and chilling of foods by immersion in aqueous fluids maintained at low temperatures. Solid foods were assumed as a porous media with an occluded solution. Three phases were considered, the rigid solid matrix, the liquid phase, and the ice phase. Transport equations for a continuous media were applied to each phase. The averaging-volume method developed by Whitaker (1977) was used for obtaining comprehensive equations to predict solute concentration and temperature as a function of space and time. The resultant set of equations is a nonlinear problem that can be solved numerically.

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**Nomenclature**

| | |
|---|---|
| $a$ | characteristic dimension (m) |
| $A$ | variable defined in Eq. (21) |
| $b$ | coefficient of Eqs. (37) and (39) |
| $c$ | coefficient of Eqs. (37) and (39) |
| $C_{P\text{eff}}$ | effective specific heat ($\text{J kg}^{-1}\,°\text{C}^{-1}$) |
| $C_{P\text{f}}$ | specific heat of the completely frozen food ($\text{J kg}^{-1}\,°\text{C}^{-1}$) |
| $C_{P\text{u}}$ | specific heat of the unfrozen food ($\text{J kg}^{-1}\,°\text{C}^{-1}$) |
| $d$ | coefficient of Eqs. (37) and (39) |
| $\mathscr{D}$ | diffusion coefficient ($\text{m}^2\,\text{s}^{-1}$) |
| $D_{\text{eff}}$ | effective diffusion coefficient for the solute ($\text{m}^2\,\text{s}^{-1}$) |
| $e_1$ | total initial mass fraction of freezable water |
| $F$ | variable defined in Eqs. (23), (48), (49), and (67)–(69) |
| $h$ | enthalpy per unit mass ($\text{J kg}^{-1}$) |
| $h_{\text{c}}$ | heat transfer coefficient ($\text{W m}^{-2}\,°\text{C}^{-1}$) |
| $\Delta H_0$ | latent heat of fusion of ice ($\text{kJ kg}^{-1}$) |
| $k_{\text{eff}}$ | effective thermal conductivity ($\text{W m}^{-1}\,°\text{C}^{-1}$) |
| $k_{\text{f}}$ | thermal conductivity of the completely frozen food ($\text{W m}^{-1}\,°\text{C}^{-1}$) |
| $k_{\text{u}}$ | thermal conductivity of the unfrozen food ($\text{W m}^{-1}\,°\text{C}^{-1}$) |
| $\langle \dot{m} \rangle$ | mass rate of water solidification ($\text{kg m}^{-3}\,\text{s}^{-1}$) |
| $N$ | number of grid segments |
| $p$ | 1, 2, and 3 for rectangular, cylindrical and spherical coordinate systems, respectively |
| $Q$ | variable defined in Eqs. (19), (45), (46), and (63)–(65) |
| $S$ | variable defined in Eq. (17) |
| $t$ | time (s) |
| $t_{\text{c}}$ | time that takes the geometric center to reach −5 °C (s) |
| $T$ | temperature (°C) |
| $T_0$ | freezing point of pure water (°C) |
| $T_{\text{f}}$ | initial freezing point (°C) |
| $T_{\text{ref}}$ | reference temperature (°C) |
| $x$ | distance along $x$-axis (m) |
| $y$ | distance along $y$-axis (m) |
| $z$ | distance along $z$-axis (m) |

*Greek symbols*

| | |
|---|---|
| $\varepsilon$ | volume fraction |
| $\eta$ | variable defined in Eq. (14) |
| $\rho$ | density ($\text{kg m}^{-3}$) |
| $\rho_{2\,\text{ave}}$ | average solute concentration in the solid at the immersion time $t_{\text{c}}$ ($\text{kg m}^{-3}$) |
| $\tau$ | tortuosity |
| $\langle \psi \rangle$ | spatial average of a function $\psi$ |
| $\langle \psi_\delta \rangle_\delta$ | phase average of a function $\psi_\delta$ |
| $\langle \psi_\delta \rangle^\delta$ | intrinsic phase average a function $\psi_\delta$ |

*Subscripts/superscripts*

| | |
|---|---|
| 0 | at initial time |
| 1 | water |
| 2 | solute |
| $i$ | at $i$th node |
| $j$ | at $j$th node |
| $k$ | at $k$th node |
| $n$ | at $n$th time level |
| $x$ | in the $x$ direction |
| $y$ | in the $y$ direction |
| $z$ | in the $z$ direction |
| $\alpha$ | ice phase |
| $\beta$ | liquid phase |
| $\infty$ | at the bulk immersion solution |

### 2.1. One-dimensional geometries

Table 1 shows the equations to describe heat and mass transfer during immersion chilling and freezing of foods considering a 1-D geometry (Zorrilla & Rubiolo, 2004). The dominium $0 \leqslant x \leqslant a$ represents a 1-D region where "$a$" may be the half thickness of an infinite slab, the radius of an infinite cylinder, or the radius of a sphere.

In the heat balance equation (1), enthalpy is the primary dependent variable while temperature is the secondary dependent variable (Mannapperuma & Singh, 1989). The enthalpy method is used to solve phase-change problems in situations in which the material solidification or melting takes place over an extended range of temperatures (Ozisik, 1994). A relation between enthalpy and temperature should exist to make this formulation meaningful.

The mass transfer phenomena—Eqs. (2)–(4)—basically take into account the change in the solute concentration because of the diffusion process from the immersion solution and because of the ice formation during the freezing process. Eqs. (5) and (7) are the symmetry conditions at the center of the solid food. Eq. (6) is a boundary condition of convective type while Eq. (8) is a boundary condition of fixed variable type. This last condition is acceptable for mass transfer because diffusion in the solid is usually rate-controlling in this type of processes (Schwartzberg & Chao, 1982). Eqs. (9)–(13) are the initial conditions, assumed as known values through out the food.

The system of equations (1)–(13) can be solved numerically. A finite difference method based on the control-volume approach simplifies the numerical solution (Mannapperuma & Singh, 1989). At the start, a grid