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Mathematical analysis of microwave heating process

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Abstract

The use of microwaves in the food industry is attributed to the lower time needed to increase the temperature of foodstuffs compared to the traditional heating methods. However, the heating is not uniform and the products show hot and cold spots. In order to analyze the behavior of microwaved foods a mathematical method was developed solving the unsteady state heat transfer differential equations. The model was applied to large systems for which Lambert's law is valid because it leads to similar results as Maxwell equation. It takes into account variable thermal and electromagnetic properties. The numerical solution was developed using an implicit finite difference method in one dimensional systems (sphere, infinite cylinder and slab) and an alterning direction method in two- and three-dimensional conditions (finite cylinders and brick shaped products). It allows to predict temperature profiles and heating times. The model was validated with own data of mashed potato and meat products and with experimental data from literature obtained with agar gel, sodium alginate gel and whole potato.

Keywords: Microwave heating; Mathematical modelling; Temperature prediction

1. Introduction

The use of microwaves in food heating has been mainly caused by changes in some consumer attitudes and lifestyles towards ‘‘ready to eat’’ or ‘‘quick cooking’’ meals. For these purposes, microwaves offer fast temperature rise in the foods owing to their capacity to generate heat energy inside the food, without requiring any medium as vehicle for heat transfer. A low thermal conductivity product may quickly reach high temperatures, and this does not occur in conventional heating.

There exist, however, some problems related to temperature distribution inside the product which must be accounted for. An important one is the appearance of hot spots in several zones, depending on product geometry. This phenomenon has been analyzed by many authors (Ayappa, Davis, Crapiste, Davis, & Gordon, 1991; Ayappa, Davis, Davis, & Gordon, 1992) because it has become one of the major drawbacks for application at domestic or industrial level. For instance, this uneven distribution produces dry and burned zones in products where in other zones, the minimum temperature required for processing is not reached (Sale, 1976). This worsens during sterilization since, being the method unable to guarantee lethality temperatures for microorganisms in all zones, it requires overheating in some zones to conduct a safe process thus causing quality losses.

To characterize this temperature distribution within the products, various authors have examined microwave heating and cooking from a mathematical modelling standpoint, since solutions of the governing differential equations provide a description of the process being studied.

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Nomenclature

A	area, m^2
b	index indicating surface position
C_p	specific heat, J/(kg °C)
d	distance from surface, m
GI	geometric index
h	heat transfer coefficient, W/(m^2 °C)
i	position index
k	thermal conductivity, W/(m °C)
L	half-thickness or radius, m
n	time index
P	power, W
P_0	surface power, W
Q	volumetric heat generation, W/(m^3)
t	time, s (or min)
T	temperature, °C
V	volume, m^3
W	weight, g
x	spatial coordinate, m
Δx	spatial increment, m
Δt	time increment, s

Greek symbols

α	density, kg/m^3
ε	dielectric constant (dimensionless)
ε''	dielectric loss (dimensionless)
α	attenuation factor, m^{-1}
λ	wavelength, m

Subscripts

a	air
ini	initial
inc	incident
t	total

Superscript

f	fictitious

Generally, two numerical schemes are selected to solve the equations describing the process: finite differences and finite elements. In this regard, Ohlsson and Bengston (1971) have studied one-dimensional heating and solved heat transfer equations for slab geometry using finite differences. They calculated internal temperatures in a meat block heated by microwaves. Swami (1982) has developed a finite difference model to describe heating of cylindrical and rectangular pieces of gels with high water and NaCl content.

In turn, several researchers have chosen the finite element method to solve differential equations for heating. Chen, Singh, Haghighi, and Nelson (1993) have employed finite elements with the Galerkin formulation for predicting potato heating in cylindrical geometry. Zhou, Puri, Anantheswaran, and Yeh (1995) have developed a three-dimensional finite element model to predict temperature and concentration profiles within foods heated by microwaves. Ni and Datta (2002) solved a model taking into account energy and mass transfer during heating. Datta and Anantheswaran (2001) have included recent developments in modelling of microwave heating process.

Regardless of the selected numerical scheme, the Maxwell equations are generally used to describe product behavior. They govern radiation propagation in a dielectric medium but, owing to their complex formulation, an approximation is used which considers an exponential decay of microwave energy absorption inside the product as prescribed by the Lambert's law. Ayappa et al. (1991) have compared numerical model predictions using Maxwell and Lambert laws to represent power in slabs. They obtained a critical thickness above which the use of Lambert approximation is valid and showed that the two formulations predicted identical power profiles for slabs thicker than 2.7 times the penetration depth. Similar results were reported by Barringer, Davis, Gordon, Ayappa, and Davis (1995) that compared predictions by the individual and combined models during heating of gel samples.

The objectives of the present work were:

(1) To develop a generalized mathematical model for different geometries based on the solution of the unsteady state microscopic energy balance capable of predicting temperature profiles in microwave heated foods; this will permit hot and cold spots to be located. The model takes into account temperature-dependent thermal and electromagnetic properties; it is applied to large samples, commonly handled in industrial application, allowing the use of Lambert's law to describe the internal generation term representing absorption of electromagnetic energy.
(2) To experimentally verify numerical predictions with experimental data obtained in our laboratory as well as with published information.
(3) To compare the effect of geometry on temperature profiles developed during microwave heating.

2. Theory

A mathematical model was proposed to predict temperatures during microwave food heating, taking into