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Analysis of some flexible families of distributions for estimation of wind speed distributions

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A R T I C L E I N F O

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A B S T R A C T

The wind speed distribution is the basis for the assessment of wind energy potential needed for the design of wind farms. Therefore, the proper specification of the wind speed distribution is of special importance. In this study, we propose for the first time two flexible families of distributions, the skewed generalized error and skewed *t*, for the description of the wind speed distribution. These families both are very flexible enough to accommodate the shape of the wind speed data and include some well-known distributions as special cases. Also we evaluate the performance of these flexible families relative to the widely-used Weibull distribution by using wind speed data measured in various regions of Turkey. The results indicate that these flexible families of distributions provide substantial improvement over the Weibull distribution in estimating wind speed distribution and wind power density distribution. Thus, the skewed generalized error and skewed *t* distributions can be alternatively used for assessment of wind energy potential.

1. Introduction

The wind speed probability distribution for a certain location is crucial in determining the performance of energy conversion systems. When the wind speed distribution is determined, the wind power density distribution can easily be obtained accordingly. For this reason, the proper specification of the wind speed distribution is of special importance in the assessment of wind energy potential. The power density distribution (Pdd) based on the probability density function (pdf) for wind speed, $f(v)$, can be presented by following formula :

$$Pdd_f = \frac{1}{2}\rho A v^3 f(v), \tag{1}$$

where v shows wind speed, ρ is air density and A is the wind turbine blade sweep area. The power density based on $f(v)$ is calculated as follows:

$$P_f = \frac{1}{2}\rho A \int_0^\infty v^3 f(v) dv. \tag{2}$$

In the literature, the Weibull is most widely-used distribution to estimate wind energy potential because of its flexibility and easy computation. Also, it is used as an accepted wind speed distribution for some wind energy programs. Therefore, a number of studies have estimated the wind energy potential of the specified region by means of the Weibull distribution [1–13].

The pdfs of the Weibull distributions with two and three parameters (WD2 and WD3), i.e. $f_{WD2}(v)$ and $f_{WD3}(v)$, are respectively given as follows:

$$f_{WD2}(v) = \frac{\theta_2}{\theta_1}\left(\frac{v}{\theta_1}\right)^{\theta_2-1} e^{-\left(\frac{v}{\theta_1}\right)^{\theta_2}}, \quad v > 0, \tag{3}$$

and

$$f_{WD3}(v) = \frac{\theta_2}{\theta_1}\left(\frac{v-\theta_0}{\theta_1}\right)^{\theta_2-1} e^{-\left(\frac{v-\theta_0}{\theta_1}\right)^{\theta_2}}, \quad v > \theta_0, \tag{4}$$

where θ_0, θ_1, and θ_2 are the location, scale and shape parameters of the Weibull distributions, respectively.

The cumulative distribution functions (cdfs) of the WD2 and WD3, i.e. $F_{WD2}(v)$ and $F_{WD3}(v)$ are respectively presented as follows:

$$F_{WD2}(v) = 1 - e^{-\left(\frac{v}{\theta_1}\right)^{\theta_2}}, \quad v > 0, \tag{5}$$

and

$$F_{WD3}(v) = 1 - e^{-\left(\frac{v-\theta_0}{\theta_1}\right)^{\theta_2}}, \quad v > \theta_0. \tag{6}$$

Since the parameters of the Weibull distribution are used to inference results about wind energy potential, their estimators have received great interest in the literature [14–18]. In order to find the best method for the Weibull distribution, a lot of

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Nomenclature

v	wind speed (m/s)	A	wind turbine blade sweep area (m^2)
v_i	ith observed wind speed data	ρ	air density (kg/m^3)
n	number of all observed wind speed data	Pdd_f	wind power density distribution (W s/m)
θ_0	location parameter of the distribution	N	number of the wind speed class
θ_1	scale parameter of the distribution	x_j	jth observed probability $j = 1, \ldots, N$
θ_2	first shape parameter of the distribution	y_j	jth predicted probability calculated from a special distribution $j = 1, \ldots, N$
θ_3	second shape parameter of the distribution	z_j	jth computed value from the correlation equation for the same value of x_i
$f_{WD2}(v)$	Weibull probability density function (pdf) with two parameters	k	number of parameters of the distribution
$f_{WD3}(v)$	Weibull pdf with three parameters	R^2	coefficient of determination
$f_{STD}(v)$	skewed t pdf	RMSE	root mean square error
$f_{SGED}(v)$	skewed generalized error pdf	KS	Kolmogorov–Smirnov
$F_{WD2}(v)$	Weibull cumulative distribution function (cdf) with two parameters	$-\ln L$	negative of the natural logarithm of the likelihood function
$F_{WD3}(v)$	Weibull cdf with three parameters	AIC	Akaike information criterion
$F_{STD}(v)$	skewed t cdf		
$F_{SGED}(v)$	skewed generalized error cdf		

estimators have been proposed and compared each other. It was found that the maximum likelihood is the recommended one due to statistical properties [14–16]. Thus, the maximum likelihood (ML) method is used to estimate the parameters of the Weibull distribution in this study.

On the other hand, although some advantages of the Weibull distribution over the other distributions used in wind energy field are shown in various studies [1,19], the Weibull distribution does not exhibit good performance for all wind types encountered in nature such as low and high wind speed [19–26]. Hence, in recent years, various distributions as an alternative to the Weibull distribution have been proposed for the description of the wind speed distribution [19–29]. For instance, [20–25] introduce the distributions derived from the entropy optimization principles. [26,27] use the mixture form of the Weibull and the certain distributions. On the other hand, [28–32] discuss the use of various statistical distributions for the wind speed data. In conclusion, the mentioned studies emphases that the Weibull distribution exhibits a poor fit to wind speed data when compared with the distribution with more parameters. Therefore, in this study, we provide for the first time, two flexible families of distributions, i.e. the skewed generalized error distribution (SGED) and skewed t distribution (STD), for the description of the wind speed distribution. These families both are very flexible enough to accommodate the shape of the wind speed data and nest some well-known distributions as special cases [33,34]. We examine the suitability of these families of distributions, the SGED and STD as a wind speed distribution by using wind speed data measured in various regions of Turkey. Moreover, we compare the performance of the SGED and STD relative to the widely-used Weibull distribution by using the five model selection criteria. The analysis reveal that the SGED and STD, particularly the SGED, can provide better results than the WD2 and WD3 in estimating wind speed distribution and the power density distribution.

Considering all these issues, the study is organized as follows: Section 2 introduces the families of distribution, i.e. the SGED and STD. Section 3 provides model selection criteria. Section 4 presents the calculation and analysis concerning the SGED and STD. Section 5 concludes the study with the obtained results and some suggestions for further research.

2. Some families of distributions for wind speed distributions

The Weibull distribution has been widely accepted and recommended probability distribution to model the wind speed data. However, it can sometimes be inadequate to model all wind speed data encountered in nature. In the following, we present two alternative flexible families of distributions that may be used as models for possibly wind speed data.

2.1. Skewed generalized error distribution

The first flexible family of distributions we propose as a reasonable wind speed distribution is the skewed generalized error distribution (SGED) suggested by [35,36] and its pdf and cdf are defined by Eqs. (7), (8), respectively:

$$f_{SGED}(v) = \frac{\theta_2 e^{-\left(\frac{|v-\theta_0|}{[\theta_1(1+\mathrm{sign}(v-\theta_0)\theta_3)]}\right)^{\theta_2}}}{2\theta_1 \Gamma\left(\frac{1}{\theta_2}\right)}, \quad (7)$$

and

$$F_{SGED}(v) = \int_{-\infty}^{v} \frac{\theta_2 e^{-\left(\frac{|t-\theta_0|}{[\theta_1(1+\mathrm{sign}(t-\theta_0)\theta_3)]}\right)^{\theta_2}}}{2\theta_1 \Gamma\left(\frac{1}{\theta_2}\right)} dt, \quad (8)$$

Table 1
The formulas of model selection criteria.

Criteria	Formulas
R^2	$1 - \sum_{j=1}^{N}(y_j - x_j)^2 \Big/ \sum_{j=1}^{N}(y_j - z_j)^2$
RMSE	$\left(\sum_{j=1}^{N}(y_j - x_j)^2 \Big/ N\right)^{1/2}$
KS	$\max_{1 \leqslant i \leqslant n}(F(v_i) - (i-1)/n, i/n - F(v_i))$
$-\ln L$	$-\ln\left(\prod_{i=1}^{n} f(v_i)\right)$
AIC	$-2\ln\left(\prod_{i=1}^{n} f(v_i)\right) + 2k$

Table 2
Details of the regions for which the wind speed data were analyzed.

Stations	Latitude	Longitude	Years of wind data available	Height of ground above sea level (m)
Amasra	41°74′	32°38′	2004–2006	12
Bandırma	40°34′	27°97′	2002–2006	63
Susehri	39°74′	37°01′	2002–2006	1050
Yunak	37°87′	32°48′	2002–2006	1460