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# Composites with auxetic inclusions showing both an auxetic behavior and enhancement of their mechanical properties

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ABSTRACT

Composite materials made of auxetic inclusions and giving rise overall to negative Poisson's ratio are considered, adopting a two-steps micromechanical approach for the calculation of their effective mechanical properties. The inclusions consist of periodic beam lattices, whose equivalent mechanical properties are calculated by a discrete homogenization scheme in a first step. The hexachiral and hexagonal reentrant lattices are considered as representative of the two main deformation mechanisms responsible for auxeticity. In a second step, the equivalent properties of the composite are calculated from numerical homogenization using the finite element method. It is shown that both an auxetic behavior and enhanced moduli can be obtained for not too slender micro-beams.



## 1. Introduction

In recent years, many attempts to conceive or produce new auxetic materials and structures have led to the identification of structures endowed with negative Poisson's ratio, including metals having a cubic unit cell when loaded in the (1 1 0) direction [1], silicates [2–9], or zeolites [6–9]. However, relatively few composites with negative Poisson's ratio have been manufactured and characterized; only recent works present auxetic composites made of intrinsically auxetic phases [10], in this last work composites of low modulus made from an auxetic yarn in a woven textile.

Auxeticity (a denomination adopted to characterize materials and structured having an overall negative Poisson's ratio) has now been identified in a wide range of man-made materials, including foams, liquid crystalline polymers and micro/nano-structured polymers [11–16]. Auxetic materials present some unique properties in comparison to common materials [17], since they show enhanced mechanical properties such as hardness, indentation, shear and fracture resistance. Some of the viscoelastic properties and the deformation behavior are also shown to be enhanced by auxeticity [18]. Thanks to static and free-vibration simulations of sandwich beams with different core cellular materials, Scarpa et al. [19] obtained both an enhanced stiffness per unit weight and increased modal loss factors, using two-phase cellular solids with a re-entrant skeleton. It is accordingly thought that composites incorporating auxetic inclusions have a great potential in terms of enhanced properties.

This work is centered on the concept of auxetic composites made of an embedded auxetic inclusion into an elastic matrix. The main novelty advocated in this contribution is the consideration of architectured inclusions having a lattice topology, leading to a negative Poisson's ratio of the composite. The auxetic behavior is related to a very specific microstructure or nanostructure endowed with specific deformation mechanisms: for instance, the re-entrant honeycomb [20,21] has been identified as one typical microstructure exhibiting the NPR (this shortcut for negative Poisson's ratio will be used here and in the sequel, with PR meaning Poisson's ratio). The chiral honeycomb structure [22], the hexachiral, tetra-antichiral and rotachiral lattices [23], are further typical well-known architectured model materials exhibiting the NPR effect.

In the present paper, the theoretical construction of new classes of auxetic composite materials is considered, based on ellipsoidal and circular inclusions endowed with a negative Poisson's ratio [24]. The inclusions have a discrete topology consisting of a network of beams, and the matrix behavior is here restricted to be elastic and isotropic. A few researchers have focused on this idea and methodology by considering virtual or idealized NPR's inclusion [25–27], without taking care of whether or not such a virtual inclusion could be obtained in reality and how, especially in terms of a corresponding microstructure.

The main objective of this work is to provide a quantitative understanding of the impact of the architecture (lattice topology) and micromechanical properties of the inclusion on the overall mechanical properties of the composite, and to develop appropriate and accurate micromechanical models that can be used in a predictive manner.

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As a first step, the discrete asymptotic homogenized technique (the short cut DAH will be employed in the sequel) will be involved to derive the expressions (in closed form in the considered linear framework) of the equivalent moduli of architectured inclusion materials (the last coinage will be used to denote a micro-structure consisting of a quasi periodical array of microscopic beams), vs. the geometrical and mechanical micro-parameters of the underlying lattice. In a second step, a numerical homogenization technique based on the FE method will be employed for the computation of the overall properties of the composite material.

This contribution is organized as follows: a survey of the DAH method and the determination of the effective properties of architectured inclusions shall be exposed in Section 2. The overall properties of the resulting composite material and numerical results will be obtained in Section 3 from numerical homogenization. A summary of the main results is presented in the conclusion (Section 4).

Regarding notations, tensors are denoted using boldface symbols.

## 2. Effective properties of architectured inclusions

The overall properties of architectured lattices or systems have been investigated thanks to several homogenization techniques, amongst which discrete homogenization proves useful when considering discrete structures from the onset, such as beam lattices. The DAH method is a mathematical technique to derive the equivalent continuous medium behavior of a quasi periodical discrete structure made of the repetition of an elementary basic cell. This technique is inspired from the homogenization of periodic media developed thirties years ago by Sanchez-Palencia [28], Bakhvalov and Panasenko [29], and more recently applied by Warren and Byskhov [30] and Mourad et al. [31]. It has also been combined with the energy method by Pradel and Sab [32] to perform the homogenization of discrete media.

The method allows calculating the effective properties of such beam lattices endowed with a periodic microstructure [33]; the principal assumptions are the quasi periodical nature of the lattice and the small dimension of the period compared to the size of the macroscopic domain. A unit cell can thus be defined to perform the homogenization and the calculation of effective mechanical properties of the original lattice [33]. Recently, the DAH technique has been used in [34] to calculate the overall properties of periodic media including micropolar effects, and Assidi et al. [35] derived the equivalent mechanical properties of cellular biological structures which were shown to obey such a micropolar behavior for certain topologies of the unit cell.

The DAH technique consists by essence in assuming asymptotic series expansions of both the node displacements, tension and external forces as successive powers of a small parameter labeled $\varepsilon$, defined as the ratio of a characteristic length of the unit cell of analysis to a characteristic length of the lattice structure. Those expansions are then inserted into the equilibrium equation, conveniently expressed in weak form. The balance equation of the nodes, the force–displacement relations and the moment-rotation relations of the beams are developed by inserting those series expansions and by using Taylor's series expansion of finite differences. The discrete sums are finally converted in the limit of a continuous density of beams into Riemann integrals, thereby highlighting continuous stress and strain measures. The calculations have been completed for a quite general truss and the results give a general and closed form expression of elastic properties in the linear framework. For more details about technical aspect, one can refer to the non-exhaustive list of recent publications devoted to the subject [37,38].

The homogenized constitutive law is presently constructed from a simulation code written in symbolic language (MAPLE), using as an input the geometry of the lattice (inclusion) and the microbeams tensile and flexural rigidities, and delivering as an output the homogenized mechanical properties (classical and micropolar moduli).

The two lattices considered in this study and giving rise to an auxetic behavior are pictured on Fig. 1, in terms of the topology of the generating unit cell (the vectors $\mathbf{Y}_1, \mathbf{Y}_2$ are the translation vectors that generate the whole lattice by periodicity in the plane); they each illustrate one of the two main mechanisms responsible for auxeticity.

The re-entrant hexagonal lattice exhibits a negative Poisson's ratio due to the expansion of the lateral side of the unit cell (due to flexion of the microbeams) when a tension is applied on the other side (Fig. 2a). In the rolling up mechanism exemplified by the hexachiral lattice, beams acting as ligaments interlink circular structures that rotate under the effect of the tension acting on the ligaments (Fig. 2b).

The effective mechanical properties of those two lattices are given in closed form for both lattices as [36] (the superscript 'Hom' thereafter stands for the homogenized behavior).

The hexachiral lattice: this is an isotropic lattice, whose effective modulus and Poisson's ratio are successively given by the following lengthy expressions:

$$E^{\mathrm{Hom}} = \frac{96}{47} \times \frac{(139\sqrt{3}-71)(-575-564\eta^2+44\sqrt{3}\eta^3)E_s}{-19729-224148\eta^2+16908\sqrt{3}\eta^2-54048\eta^4+2973\sqrt{3}}. \tag{1}$$

$$\nu^{\mathrm{Hom}} = \frac{1}{647} \times \frac{(74\sqrt{3}-617)(-20689+76300\eta^2+11932\sqrt{3}\eta^3-62112\eta^4+1257\sqrt{3})}{-19729-224148\eta^2+16908\sqrt{3}\eta^2-54048\eta^4+2973\sqrt{3}} \tag{2}$$

with $\eta = t/L$ and $E_s$ respectively the slenderness ratio (ratio of the beam thickness to length (L)) and the Young modulus of the beams. Those expressions are exact in the considered linear framework, and may be simplified for slender beams ($\eta$ is small).

The re-entrant hexagonal lattice: this is an anisotropic lattice, with effective moduli and Poisson's ratio successively given vs. the angle $\theta$ and the slenderness ratio $\eta$ by

$$\begin{cases} E_{11}^{\mathrm{Hom}} = E_S \frac{\eta^3 \cos\theta}{(1+\sin\theta)(\eta^2\cos^2\theta-\cos^2\theta+1)}; \\ E_{22}^{\mathrm{Hom}} = -E_S \frac{\eta^3(\sin\theta+1)}{\cos\theta(-\cos^2\theta+\eta^2\cos^2\theta-3\eta^2)}; \\ G_{12}^{\mathrm{Hom}} = \frac{E_S\eta^3(\sin\theta+1)\cos\theta}{3\cos^2\theta+2\eta^2\sin\theta+2\eta^2-\eta^2\cos^2\theta} \end{cases} \tag{3}$$

$$\begin{cases} \nu_{12}^{\mathrm{Hom}} = \frac{(\eta^2-1)\sin\theta(\sin\theta-1)}{(\eta^2\cos^2\theta-\cos^2\theta+1)}; \\ \nu_{21}^{\mathrm{Hom}} = \frac{(\eta^2-1)\sin\theta(\sin\theta-1)}{(-\cos^2\theta+\eta^2\cos^2\theta-3\eta^2)} \end{cases} \tag{4}$$

Note that since the equivalent continuum is of Cosserat type, there are further micropolar moduli, which are however not listed, as they are not required in the present analysis. Furthermore, they do not play a role in the determination of the tensile moduli and Poisson's ratio, but may modify the effective shear moduli. Let notes that the calculation of the overall effective properties of the embedded inclusion, by the DAH technique, is based on the micropolar media. The importance of micropolar effects at the macroscopic scale can be assessed by a comparison of the characteristic bending length of the micropolar continuum (a byproduct of the elasticity constants of this medium) to the beam and lattice