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DSC analysis of a simply supported anisotropic rectangular plate

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ABSTRACT

The discrete singular convolution (DSC) algorithm is used to analyze the deflection and free vibration behavior of a simply supported anisotropic rectangular plate. A novel approach is proposed to solve the difficulty in using DSC to handle the simply supported boundary conditions with bending–twisting coupling. DSC results are presented for bending under distributed load and a center concentrated load, and natural frequencies of flexural vibrations. It is shown that the DSC with proposed method to apply the simply supported boundary conditions yields very accurate results as compared to exact solutions or results obtained by methods of differential quadrature and finite element with fine meshes. It is also verified that neglecting the bending–twisting coupling in applying the simply supported boundary conditions may result incorrect solutions, especially for the bending analysis of anisotropic plates.

1. Introduction

The demand for improved structural efficiency in high performance air vehicles has resulted in consideration of fiber-reinforced composite materials as the plate material [1]. Since the rectangular plate is a common structural element, therefore, it is important for designers to understand the anisotropic mechanical behavior of such components.

It is observed that the bending–twisting coupling of the boundary conditions makes a closed form solution be very difficult to obtain even for a rectangular plate simply supported along four edges. Therefore, various approximate or numerical methods, such as Rayleigh-Ritz method, Galerkin method, finite element method, finite difference method, and differential quadrature method [2], are employed for obtaining solutions. Although the assumed-mode methods such as Rayleigh-Ritz and Galerkin methods need less computational effort as compared to the numerical methods (finite element and finite difference), however, it is not an easy task to select the test functions satisfying all boundary conditions with bending–twisting coupling. If the test functions are only satisfied the geometrical boundary conditions, the rate of convergence of the solution obtained by Rayleigh-Ritz method may be low for analysis of anisotropic plates with all edges simply supported [3–5]. Even worse, the results for simply supported symmetrically laminated composite plates, obtained by Rayleigh-Ritz method with double sine series to describing the transverse deflection, do not converge to the correct solutions with increasing series order [6]. Therefore, it seems necessary to seek some alternative efficient methods.

The differential quadrature (DQM) has been shown one of the alternative efficient methods for analyzing anisotropic rectangular plates [2]. Due to its compactness and computational efficiency, the DQM is more attractive than the Rayleigh-Ritz method for analysis of anisotropic composite plates. It is shown [7], however, that the conventional DQM cannot efficiently analyze a plate subjected to a concentrated load. Therefore, investigations on some other alternative efficient methods seem still necessary.

The discrete singular convolution (DSC) algorithm, proposed by Wei [8], is one of the other alternative efficient methods. It is shown [9] that similar solution accuracy as to DQM can be achieved for isotropic plates. The DSC has been successfully used in solving some challenge problems such as vibration of plates with irregular internal supports [10] and mixed boundary conditions [11], higher-order modes vibration [12], vibration and stability analysis of arbitrary straight-sided quadrilateral plates [13], and static and free vibration of composite plates [14–16].

It is noticed that although very accurate predictions have been obtained for both isotropic and orthotropic plates by DSC with small number of grid points [14], but much less accurate predictions of natural frequency are obtained for symmetrically laminated composite plates. The DSC predictions are even higher than the upper bound solutions given by Leissa and Narita [17]. The reason is that the method of anti-symmetric is used for applying the simply supported boundary conditions thus the bending–twisting coupling is omitted. Although the maximum relative difference between DSC data and Leissa's upper bound solutions is within 1.3%, however, one cannot conclude that the DSC can be reliably used in the static and vibration analysis of simply supported composite plates, since $E_1/E_2 = 2.45$ and the anisotropy is not pronounced for the case considered in [14], where E_1 and E_2 are the elasticity modulus in the fiber direction and transverse

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Nomenclature

DQM	differential quadrature method
E_1	the modulus of elasticity in the fiber direction
E_2	the transverse modulus of elasticity
FEM	finite element method
a	length of a rectangular plate
b	width of a rectangular plate
N_x	total number of grid points in the x-direction
N_y	total number of grid points in the y-direction
N	total number of grid points in the x- and y-direction
DSC-LK	DSC with the non-regularized Lagrange's delta sequence kernel
$w(x)$	function of x
a, b	the length and width of a rectangular plate
x, y	the Cartesian coordinates
x_k	coordinates of the uniformly distributed grid points
$w^{(n)}(x)$	the nth order derivative of $w(x)$
$2M+1$	the computational bandwidth
$\widetilde{A}_{0k}$	"weighting coefficients" of the first order derivative at $x = 0$
$\widetilde{B}_{0k}$	"weighting coefficients" of the second order derivative at $x = 0$
$\widetilde{C}_{0k}$	"weighting coefficients" of the third order derivative at $x = 0$
$\widetilde{D}_{0k}$	"weighting coefficients" of the fourth order derivative at $x = 0$
FPs	fictitious points
$w'(0)$	the first order derivative of $w(x)$ at x=0
$w''(0)$	the second order derivative of $w(x)$ at $x = 0$
$w'''(0)$	the third order derivative of $w(x)$ at x=0
A_{ik}	"weighting coefficients" of the first order derivative
B_{ik}	"weighting coefficients" of the second order derivative
C_{ik}	"weighting coefficients" of the third order derivative
D_{ik}	"weighting coefficients" of the fourth order derivative
$\overline{D}_{ij}$	effective plate bending or twisting stiffness
$w(x, y)$	deflection
h	plate thickness
$q(x, y)$	the distributed load
A^x_{ik}	"weighting coefficients" of the first order derivative w.r.t. x
B^x_{ik}	"weighting coefficients" of the second order derivative w.r.t. x
C^x_{ik}	"weighting coefficients" of the third order derivative w.r.t. x
D^x_{ik}	"weighting coefficients" of the fourth order derivative w.r.t. x
A^y_{ik}	"weighting coefficients" of the first order derivative w.r.t. y
B^y_{ik}	"weighting coefficients" of the second order derivative w.r.t. y
C^y_{ik}	"weighting coefficients" of the third order derivative w.r.t. y
D^y_{ik}	"weighting coefficients" of the fourth order derivative w.r.t. y
DSC-A	DSC-LK with the method of anti-symmetric to eliminate the FPs
G_{12}	shear modulus
$\bar{w}$	non-dimensional deflection
D_0	bending rigidity in the fiber direction
E/E	E-glass/epoxy
G/E	Graphite/epoxy
DSC-RSK	DSC with regularized Shannon's delta kernel
DSC-LK-A	DSC-LK with the method of anti-symmetric to eliminate the FPs
DSC-RSK-A	DSC-RSK with the method of anti-symmetric to eliminate the FPs
DSC-T	DSC-LK with the method of Taylor series expansion to eliminate the FPs
DSC	the discrete singular convolution
DSC	DSC-LK with the proposed method to eliminate the FPs
PSM	the polynomial series method
Nc	the center grid point

Greek letters

Δx	uniform grid spacing in the x-direction
Δy	uniform grid spacing in the y-direction
$\delta_{\alpha,\sigma}(x - x_k)$	a collective symbol for the delta kernels of Dirichlet type
ρ	mass density
v_{12}	Poisson's ratio
ω	circular frequency
$\bar{\omega}$	non-dimensional circular frequency
$\delta(x)$, $\delta(y)$	Dirac delta functions
θ	the orientation angle of the fiber to the x axis

direction, respectively. The accuracy of DSC results would be even less for plates having strong bending–twisting coupling. Smaller deflections under transverse loads and larger free vibration frequencies would be expected due to overestimate the stiffness of the plate, as was pointed out by Stone and Chandler [6]. Perhaps this is the main reason why the static analysis by using DSC [15,16] is only for orthotropic composite plates when simply supported boundary condition is encountered. Very recently, Zhu and Wang [18] tried to solve this problem but had only a little success, although the bending–twisting has been taken into considerations. This indicates that the way to apply the boundary condition is very important in applying DSC for analysis of composite plates.

The objectives of the present paper are twofold. Firstly, it will be demonstrated that the numerical results may even converge to incorrect solutions if the bending–twisting coupling in applying the simply supported boundary conditions is omitted as was done in [14]. Secondly, a new efficient method to overcome the existing difficulty in using DSC is proposed. A variety of examples are solved by using DSC together with the proposed method for applying the simply supported boundary conditions. DSC results are compared to exact solutions and data obtained by Rayleigh-Ritz method, DQM and finite element method (FEM) for bending under distributed load and a central concentrated load, and natural frequencies of flexural vibrations.

2. Discrete singular convolution algorithm

In the DSC algorithm, many kernels are available and the commonly used kernel is the regularized Shannon kernel (called DSC-RSK in short) [10–16]. To avoid the difficulty of selecting an optimum value for the kernel parameters, the non-regularized Lagrange's delta sequence kernel, called DSC-LK [9] or sometimes simply DSC in this paper, is to be employed for obtaining solutions of anisotropic rectangular plates. Since there are no kernel parameters to be optimized, DSC-LK is simple in derivations. Besides, DSC-LK can also yield similar solution accuracy as DSC-RSK [18].

Let $\Delta x = a/(N_x - 1)$ and $\Delta y = b/(N_y - 1)$, where N_x and N_y are the total number of grid points in the x- and y-direction, a and b are the