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A high-order theory for the analysis of circular cylindrical composite sandwich shells with transversely compliant core subjected to external loads

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ABSTRACT

A new model based on the high order sandwich panel theory is proposed to study the effect of external loads on the free vibration of circular cylindrical composite sandwich shells with transversely compliant core, including also the calculation of the buckling loads. In the present model, in contrast to most of the available sandwich plate and shell theories, no prior assumptions are made with respect to the displacement field in the core. Herein the displacement and the stress fields of the core material are determined through a 3D elasticity solution. The performance of the present theory is compared with that of other sandwich theories by the presentation of comparative results obtained for several examples encompassing different material properties and geometric parameters. It is shown that the present model produce results of very high accuracy, and it is suggested that the present model, which is based on a 3D elasticity solution for the core material, can be used as a benchmark in future studies of the free vibration and buckling of circular cylindrical composite sandwich shells with a transversely compliant core.

1. Introduction

Composite sandwich plate and shell structures with transversely flexible core material, such as e.g. polymeric foams, have gained wide spread acceptance as an excellent means of obtaining lightweight structures with very high stiffness-to-weight and strength-to-weight ratios. The application area covers a wide range including aerospace, marine, wind turbine blades and even civil engineering structures, to mention but a few. To accommodate for this development, various theories for the analysis of advanced composite sandwich structures have been developed.

The exact analysis of problems involving sandwich shell structures requires the use of 3D elasticity theory. However, since the number of exact 3D elasticity solutions available is very limited, and since 3D finite element analysis of sandwich shell structures involves very large computational efforts, approximate 2D plate or shell models are being preferred in most applications.

The available approximate 2D theories can be categorized into two principal groups [1] according to the variable description of the layers of the structural assembly.

Equivalent single layer models (ESL) preserve the number of the unknown variables independent of the number of the constitutive layers in the structural assembly. Based on a higher order ESL theory Kant and Swaminathan [2] presented an analytical formulation to derive the natural frequencies of composite and sandwich plates. In ESL theories, the layered sandwich structure is analyzed as a 2D equivalent single layer by assuming that the displacements can be described in the form of continuously differentiable functions of the thickness coordinate. Although ESL models can predict global behavior of thin and some moderately thick laminates, they are not able to accurately predict the overall behavior of sandwich structures with a transversely flexible or compliant core [3].

Alternatively, in layer-wise (LW) theories each layer of the structural assembly is considered separately, by assuming (or deriving) a displacement field for each layer. LW models may be conceived as a compromise between the simplicity of the ESL theories and the complex 3D elasticity theory, but even for the LW theories the number of unknowns becomes very large as the number of layers increases.

To retain the benefits of less computational requirements of the ESL theories and the accuracy and high level of solution detail of the layer-wise theories, the so-called zig-zag theories were proposed. The discontinuity of physical/mechanical properties in the thickness direction makes inadequate those theories which were originally developed for one-layered structures. These theories are, in fact, not able to reproduce piecewise continuous

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Nomenclature

A_{mni}	extensional stiffness matrix
B_{mni}	bending-stretching coupling matrix
D_{mni}	flexural stiffness matrix
d_i	thickness of the outer and the inner face sheets
E_c, $G_{\phi c}$, G_{xc}	Young's and shear modulus for the core
h	total thickness of sandwich shell
h_c, h_f	thickness of the core and the face sheets
$i(=c,t,b)$	indices for core, outer (top) and inner (bottom) face sheets
I_{ji}	moment of inertia
j	imaginary unit
K_i, k_0, c_i	constants
L, a	length and width of shell
M_x^i, M_ϕ^i, $M_{x\phi}^i$	moment resultants
$\tilde{M}$, $\tilde{K}$	mass and stiffness matrices
m, n	wave numbers
N_x^i, N_ϕ^i, $N_{x\phi}^i$	stress resultants
$\tilde{N}_{xx}^i$, $\tilde{N}_{\phi\phi}^i$, $\tilde{N}_{x\phi}^i$	in-plane external loads
$\bar{P}$	dimensionless compressive load
q_r^i	radial distributed loads
r_i	radii of curvature for each face sheet
r_{ic}	radii of curvature at core-face interfaces
T, U	kinetic and potential energy
V	work done by applied forces
V_i	volumes of the face sheets and the core
u_i, v_i, w_i	displacements in longitudinal, circumferential and radial directions
u_{0i}, v_{0i}	mid-plane displacements in longitudinal and circumferential directions
x, ϕ, z	longitudinal, circumferential and radial coordinates
α_0	sector angle of shell
$\beta_{xx\ i}$, $\beta_{\phi\phi i}$	rotations of the face sheets
δ	variational operator
ε_{rrc}, $\gamma_{x\ rc}$, $\gamma_{\phi rc}$	radial and shear strains in the core
$\varepsilon_{xx\ i}$, $\varepsilon_{\phi\phi i}$, $\gamma_{x\phi i}$	strains in face sheets
ε_{xx0i}, $\varepsilon_{\phi\phi 0i}$, $\gamma_{x\phi 0i}$	mid-plane strains in face sheets
$\varepsilon_{xx\ i}$, $\varepsilon_{\phi\phi i}$, $\gamma_{x\phi i}$	in-plane normal and shear strains in the face sheets
$\kappa_{xx\ i}$, $\kappa_{\phi\phi i}$, $\kappa_{x\phi i}$	mid-plane curvatures in face sheets
ρ_i, ν_{ij}	mass density and Poisson's ratio
$\sigma_{xx\ i}$, $\sigma_{\phi\phi i}$, $\tau_{x\phi i}$	in-plane normal and shear stresses in the face sheets
σ_{rrc}, $\tau_{x\ rc}$, $\tau_{\phi rc}$	radial and shear stresses in the core
ω	natural frequency
$\bar{\omega}$	dimensionless natural frequency

displacement and transverse stress fields in the thickness direction [4]. The piecewise form of transverse stress and displacement fields are often described in the open literature as zigzag and interlaminar continuity, respectively. The theories which describe these two effects are referred to as zigzag theories.

Carrera [4] in his historical review paper has shown that, three independent ways of introducing zigzag theories have been proposed for the analysis of multilayered plates and shells. Lekhnitskii [5] was the first to propose a theory for multilayered structures which describe the zigzag behavior of a displacement field in the thickness direction and interlaminar equilibrium of transverse stresses. Lekhnitskii's work was originally presented as an elasticity solution for layered, cantilever beams. Apart from the method by Lekhnitskii, which was extended to plate structures by Ren [6], the other two approaches were proposed by Ambartsumian (who extended the well known Reissner–Mindlin theory to anisotropic layered plates and shells); and by Reissner who proposed a variational theorem that permits both displacement and transverse stress assumptions. Ambartsumian [7] has considered refinements of Reissner–Mindlin theory, which was originally developed for isotropic, monocoque structures, directly to make it suitable for the application to anisotropic layered plates and shells. A third approach to laminated structures was originated by two papers by Reissner [8,9] in which a mixed variational equation, namely Reissner's Mixed Variational Theorem was proposed. Reissner's Mixed Variational Theorem permits one to satisfy, completely and a priori, the C_z^0 requirements by assuming two independent fields for displacements and transverse stresses. Briefly, this theory expresses 3D indefinite equilibrium equations and compatibility equations for transverse strains in a variational form. According to Ref. [4], Lekhnitskii Multilayered Theory although very promising, has almost been ignored in the open literature. Dozens of papers have instead been presented which consist of direct applications or particular cases of the original Ambartsumian Multilayered Theory. The contents of the original works have very often been ignored, not recognized, or not mentioned in the large number of articles. Reissner Multilayered Theory seems to be the most natural and powerful method to analyze multilayered structures [4].

Kulkarni and Kapuria [10] employing a third-order zigzag theory for the study of the free vibration response of sandwich plates. Wang et al. [11] considered two different angles of rotation for the face and the core and studied the free vibration of rectangular and skew sandwich plates based on the p-Ritz method, where p-Ritz functions were constructed as the product of mathematically complete polynomials.

Some researchers have proposed so-called global–local models to provide a computational compromise between layer-wise theories and global higher order theories. Zhen and Wanji [12] used a global–local higher order theory to study the buckling response of laminated composite and sandwich plates subjected to thermal and mechanical compressive loads. Shariyat [13,14] used a generalized global–local theory to study the vibration and thermo-mechanical buckling of laminated composite and sandwich plates, by enforcing continuity requirements for all the displacement and transverse stress components across the layer interfaces.

As an alternative to the above classification of models, approximate 2D theories can be distinguished by the choice of the primary variables. If only displacements are used, the corresponding models can be referred to as "classical", and the derived structural models can be formulated on the basis of the principle of virtual displacements. If stresses are also used as unknowns, the derived models are referred to as "mixed models". Among the various mixed variational statements the Reissner's mixed variational theorem (RMVT), which was proposed in [8] and over-viewed in [15], appears very suitable for multilayered structures since only transverse stresses are added to displacements in order to fulfill the interlaminar continuity [1]. Interlaminar continuity of transverse stresses, as well as the zigzag form of displacements in the thickness plate/shell directions, are in fact easily introduced by RMVT. In particular, RMVT does not show any complicating effects when including the fundamental effects of transverse normal stresses and strains. Furthermore, RMVT furnishes transverse stresses a priori; that is, post-processing procedures are not required. Results have demonstrated that RMVT permits one to obtain a three-dimensional description of stress and strain fields of multilayer structures [15]. Dafedar et al. [16] have proposed two sets of mixed models based on layer-