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Effective elastic modulus and micro-structure damage of particle-reinforced composites

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ABSTRACT

A new analysis method of effective elastic modulus for composites has been developed by combining Eshelby's equivalent inclusion method and self-consistent method. The equations obtained can describe the evolution of debonding damage of the composites with multi-phase particles and single-phase particles. Based on the incremental relation between particles and the matrix, the incremental constitutive relations of composite, matrix, particles and voids have been developed. Numerical analysis has been conducted for Ramburg–Qsgood function incorporating with equivalent elastic modulus obtained. The constitutive equation curves for different particle volume fractions can describe the influence of debonding damage on effective elastic modulus of the composites. Numerical results of the present study have a better agreement with the experimental results.

1. Introduction

Effective elastic modulus of composite, which plays an important role in the analysis and design of composite materials, has been studied extensively. Many researches have attempted to estimate the effective elastic modulus of composite by both analytical and numerical methods. The elasto-plastic deformation of particle-reinforced composites was normally simulated using the mean-field models, where the complex stress and strain fields in each phase are replaced by their volume-averaged values, or by the finite element analysis of single-particle periodic cells.

Damage occurs during deformation because of particle cracking or particle/matrix interface decohesion. The microcracks thus formed reduce the material properties of composite. Studies have been attempted to account for nontrivial problems such as damage appearance in the material [1–5]. Niordson and Tvergaard [6], and Xue et al. [7] carried out unit-cell analysis for discontinuously reinforced composite by using a finite element method based on the strain gradient plasticity [8], and discussed the size effects of reinforcements on overall deformation behavior of composite. Nan and Clarke [9] extended the Nan-Yuan's effective medium approach [10] by introducing the particle size effects into stress–strain relation of matrix in composite and damage criterion of particles. Togho et al. [11] established an incremental damage model, which is based on Mori–Tanaka's mean field concept and can describe the evolution of debonding damage, matrix plasticity and particle size effects on the deformation and damage of composites.

In this paper, the effective bulk and shear modulus of composites have been obtained by establishing a link between two strains: the strain of matrix/particle and the strain of an equivalent material, of which the moduli are unknown. In the present work, the modified model, which is based on Eshelby's equivalent inclusion method and self-consistent method, can describe the influence of interfacial debonding damage between the particle and matrix. Numerical analyses are carried out based on the two conditions: the bulk/shear modulus for various volume fractions of particles, and the stress–strain response for composites in the cases of considering particle size effects and debonding damage. The estimation equations obtained can describe the change rule of effective elastic modulus due to the debonding damage.

2. Constitutive relation of composite with multi-phase and single-phase particles

Fig. 1a shows the composite reinforced by the multi-phase of particles, and it is in the process of damage. The composite contains intact, partially debonded and fully debonded particles. As shown in Fig. 1b, a damage model based on micro-mechanics is developed. The volume fractions of intact particles and voids are $\sum_{i=1}^{n} f_p^i$ and $\sum_{i=1}^{n} f_v^i (= f_{pt} - \sum_{i=1}^{n} f_v^i)(i = 1, \ldots, n)$, respectively. Here, f_{pt} is the initial particle volume fraction. The incremental deformation process is denoted by $\mathrm{d}f_i$. The progress of damage in the composite is described by a volume fraction of intact particles turning to a volume fraction of voids and the corresponding stress redistribution as shown in Fig. 1c and d. The composite is statically homogeneous and macroscopically isotropic before and after damage. The debonding damage process in the composite can be simulated

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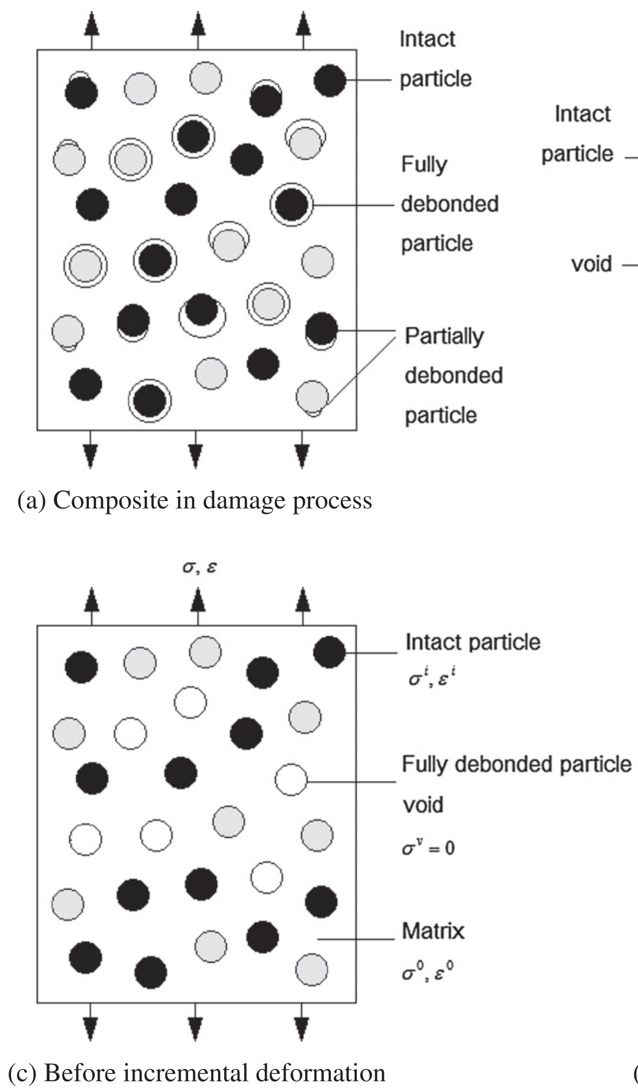

Fig. 1. Schematic illustrations of a composite in debonding damage and a composite in micro-mechanics model (multi-phase particles).

by the assumptions: Debonding of particles is controlled by a critical value of particle stress, because the interfacial stress between particles and matrix is described as a function of the particle stress [12,13]. During debonding, the stress of debonded particle is released and the site of particle is regarded as a void. A volume fraction of debonded particles turns into a volume fraction of void, and progressive damage in the composite is expressed by a decrease in an intact particle volume fraction and an increase in a void volume fraction [11]. According to the study by Togho et al. [11], the stress of intact particles in the micro-mechanics model is uniform and given by $\sigma^i_{mn}, (i = 1, \ldots, n)$, and average stresses of intact, fully-debonded and partially-debonded particles are approximately described by $\langle \sigma^i_{mn} \rangle = k\sigma^i_{mn}$, where $k = 1$ for intact particles, $k = 0$ for fully-debonded particles, and $0 < k < 1$ for partially-debonded particles depending on debonding area. The overall stress of the composite is developed as follows:

$$\begin{aligned}\sigma_{mn} &= \left(1 - \sum_{i=1}^{n} f^i_p\right)\langle \sigma^0_{mn} \rangle + \sum_{i=1}^{n} \int_0^{f^i_{pt}} k\sigma^i_{mn} \mathrm{d}f_i \\ &= \left(1 - \sum_{i=1}^{n} f^i_p\right)\langle \sigma^0_{mn} \rangle + \sum_{i=1}^{n} f^i_p \sigma^i_{mn} \end{aligned} \tag{1}$$

where $\langle \sigma^0_{mn} \rangle$ is an average stress of the matrix and

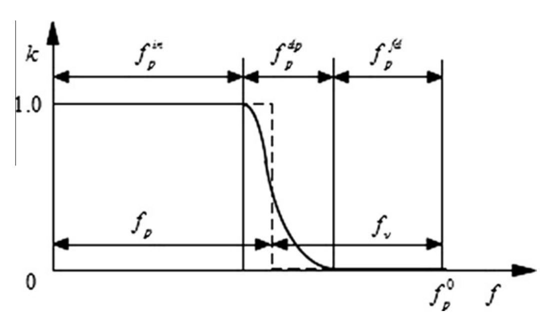

Fig. 2. Coefficient k for intact, partially dedonded, and fully debonded particles.

$$f^i_p = \int_0^{f^i_{pt}} k \mathrm{d}f_i \tag{2}$$

$$f^i_v = f^i_{pt} - f^i_p \tag{3}$$

where f^i_{pt} is an initial volume fraction of the i-phase particles, and the coefficient k is shown in Fig. 2.