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Technical Note

On using exponential basis functions for laminates modeled by CLPT, FSDT and TSDT: Further tests and results

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ABSTRACT

In this paper we present some results from the application of a mesh-free method introduced previously (Compos Struct 2011;93:3112–9 and 94:84–91) for bending analysis of laminated composite plates. This method is applicable to a wide range of bending problems without limitation in the stacking sequence of the laminated plates and the boundary conditions. Herein, two specific types of problems, having traction free boundaries, are examined and the issues related to the solution of them are addressed. Also as new benchmark problems, some more results for cross-ply and angle-ply composites are presented.

1. Introduction

A meshless boundary point method for static analysis of isotropic and anisotropic laminated composite plates has been developed in our previous work [1]. The original idea can be found in [2] and the latest applications may be seen in [3–5]. In the proposed method the generalized displacements of the plate are approximated by a series of exponential basis functions (EBFs) satisfying the governing partial differential equations (PDEs). The boundary conditions are enforced through a collocation approach on a set of boundary points. The unknown coefficients of the approximation series are obtained by a specific transformation technique that makes it possible to impose both the essential and natural boundary conditions simultaneously (see also [6–8] for other applications). The enforced boundary values are the modified forms of the real boundary conditions which are the difference of the real boundary values and those of the particular solutions on the boundary points. The method has been implemented in [3] for bending analysis of isotropic/orthotropic cross-ply laminated plates with symmetric/non-symmetric layers based on the classical plate theory (CLPT), the first order shear deformation theory (FSDT) and the third order shear deformation theory (TSDT). Therein, several benchmark plate problems with various geometries and boundary conditions have been solved to validate the method. Here we do not intend to present the history of the theories, however, the readers may refer to [9–13] for further information and latest theories.

Through our extensive investigations, we have been faced with few issues related to certain types of geometry and boundary conditions. This is mainly due to the fact that we impose the boundary conditions with the aid of a point-wise method. In this paper we shall address these issues by presenting relevant sample problems and propose simple and effective remedies to resolve them.

Here we shall focus on problems with at least one traction-free edge. The first issue is related to using shear deformation theories in solution of thin plates with traction free-edges. We shall recall a sample problem solved in [3], wherein the stress analysis of a sector plate with simple supports at two radial edges, has been considered. Here we are particularly interested in the case of traction-free circular edges not reported in [3]. We shall discuss on the number of boundary conditions used in CLPT and FSDT to propose a remedy suitable for FSDT, and consistent with CLPT, when the thickness of plate decreases.

The second issue is related to plates with kinked traction-free edges modeled with CLPT. We shall show that when using a collocation approach for boundary conditions, specific conditions is needed to be satisfied at the kinked boundaries. To illustrate the issue, we consider bending solution of quadrilateral plates, using CLPT, having adjacent traction-free edges.

The two issues mentioned above are directly related to boundary conditions in thin plate theory. Therefore we shall overview the variational formulation from which the conditions are derived. The layout of paper is as follows. In Section 2 we briefly explain the method used in [1,3]. In Section 3 we give an overview on the boundary conditions in CLPT. In Section 4 the results of the numerical examples are presented and discussed. In Section 5 we summarize the conclusions made in other sections.

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2. The methodology

In the proposed method, we split the total solution $\mathbf{u}$ into homogenous and particular solutions, as $\mathbf{u} = \mathbf{u}_h + \mathbf{u}_p$. The homogenous solution $\mathbf{u_h}$ and the particular solution $\mathbf{u_p}$ should be determined such that

$$\mathbf{L}\mathbf{u}_p = \mathbf{q} \quad \text{in} \quad \Omega \tag{1}$$

$$\mathbf{L}\mathbf{u}_h = \mathbf{0} \quad \text{in} \quad \Omega \tag{2}$$

$$\mathbf{L_B}(\mathbf{u}_h + \mathbf{u}_p) = \mathbf{L_B}\mathbf{u}_h + \mathbf{L_B}\mathbf{u}_p = \mathbf{u}_B \quad \text{on} \quad \partial\Omega \tag{3}$$

where $\mathbf{L}$ is the differential operator and $\mathbf{L_B}$ is the boundary condition operator (see [1]). Here $\mathbf{u_B}$ contains the prescribed boundary conditions on $\partial\Omega$. The difference between $\mathbf{u_B}$ and $\mathbf{L}_B\mathbf{u}_p$ yields a modified form of the boundary conditions, which should be satisfied by the homogenous part of the solution. One can find a detailed description of determination of the particular and homogenous solutions in [1] for laminated plate problems.

In summary, we assume the approximate homogenous solution as a summation of exponential basis functions (EBFs) with unknown coefficients, i.e.

$$\mathbf{u}_h = \sum_i c_i \mathbf{h}_{(\alpha_i,\beta_i)} e^{\alpha_i x + \beta_i y}, \quad \forall(x,y) \in \Omega \quad \text{and} \quad (\alpha_i, \beta_i) \in \mathbb{C}^2 \tag{4}$$

where $\mathbf{h}_{(\alpha_i,\beta_i)}$ is a vector containing the contribution of the basis function to the generalized displacement coefficients. The EBFs are chosen (see [1,2]) such that they satisfy the homogenous differential equations

$$e^{\alpha_i x + \beta_i y}(\mathbf{L}_{(\alpha_i,\beta_i)} \mathbf{h}_{(\alpha_i,\beta_i)}) = \mathbf{0}. \tag{5}$$

Upon imposing the boundary values on the edges, the unknown coefficients c_i of the approximation series are evaluated. This is performed by a boundary collocation approach on a set of boundary points. The unknown coefficients are evaluated by a discrete transformation technique. Based on this technique, the unknown coefficients can be computed by finding the projection of a vector containing basis functions' values on the vector of boundary values at the boundary points via a projection matrix.

Boundary points are chosen in a straightforward manner. Nevertheless, at the corners of the plate, one can either consider one corner point or instead use two separate and closely spaced points at the either sides of the corner. In the latter choice the imposition of the boundary values is straightforward compared with the former one. Next we overview the boundary conditions required at a traction-free corner.

3. Boundary conditions for traction-free edges in CLPT; an overview

Boundary conditions in thin plates may be defined through a variational analysis. Using an appropriate functional for the plate, here the total potential energy, one may write

$$\Pi = U + V \tag{6}$$

where U and V represent the strain energy and the potential of the external loads, respectively. In thin plates V is written as follows:

$$V = \oint_s \frac{\partial w}{\partial n} M_{nn} ds + \oint_s \frac{\partial w}{\partial s} M_{ns} ds - \oint_s w Q_n ds - \int_\Omega wp d\Omega. \tag{7}$$

In which $s \equiv \partial\Omega$, w is the plate deflection and p is the intensity of the transverse load. Also M_{nn}, M_{ns} and Q_n are, respectively the intensity of the flexural moment, the twisting moment and the shear forces acting at the point with unit vector $\mathbf{n}$ normal to the boundary. The signs in (7) are determined by considering the positive direction of the quantities (see [14]). The second term in (7) is rewritten using integral by parts and then combined with the third term to obtain

$$V = \oint_s \frac{\partial w}{\partial n} M_{nn} ds - \oint_s w\left(Q_n + \frac{\partial M_{ns}}{\partial s}\right) ds - \int_\Omega wp d\Omega. \tag{8}$$

The equilibrium state is obtained by considering the variation of (6) as

$$\delta U + \delta V = 0 \tag{9}$$

in which

$$\delta V = \oint_s \frac{\partial \delta w}{\partial n} M_{nn} ds - \oint_s \delta w\left(Q_n + \frac{\partial M_{ns}}{\partial s}\right) ds - \int_\Omega \delta wp d\Omega. \tag{10}$$

Considering all terms in (9) leads to derivation of the differential equation and the boundary conditions (this is a straightforward procedure). It is common to use the second term in (10) and define the so called Kirchhoff's shear forces as

$$V_n = Q_n + \frac{\partial M_{ns}}{\partial s}. \tag{11}$$

However, the prerequisite of such definition is the integration by parts performed in (7) to arrive at (10). From (10), one may derive the boundary conditions for a traction-free edge as

$$M_{nn} = 0, \quad V_n = 0. \tag{12}$$

It can be seen that two conditions suffice to define a traction-free boundary in thin plate theory. However, for shear deformable theories it is needed to satisfy three conditions (see [1,14]). This is the issue that we shall refer to in the first numerical example.

Now, we consider a kinked boundary at point "p" with two distinct normal unit vectors as $\mathbf{n}_p^{(-)}$ and $\mathbf{n}_p^{(+)}$. For the second term in (7) integration by parts gives

$$\oint_s \frac{\partial w}{\partial s} M_{ns} ds = [wM_{ns}]_{s_p^{(-)}} - [wM_{ns}]_{s_p^{(+)}} - \oint_s w \frac{\partial M_{ns}}{\partial s} ds \tag{13}$$

where $s_p^{(-)}$ and $s_p^{(+)}$ denote tangential coordinates of the two sides of point "p". Therefore (10) may be rewritten as

$$\delta V = \oint_s \frac{\partial \delta w}{\partial n} M_{nn} ds + \delta w \left\{ [M_{ns}]_{s_p^{(-)}} - [M_{ns}]_{s_p^{(+)}} \right\} - \int_s \delta w \left(Q_n + \frac{\partial M_{ns}}{\partial s}\right) ds - \int_\Omega \delta wp d\Omega. \tag{14}$$

It can be seen that the second term acts as a concentrated force at the kinked boundary (see also [14]). If "p" is located at a traction-free boundary, then the boundary conditions are defined as

$$M_{nn} = 0, \quad V_n = 0, \quad \left\{ [M_{ns}]_{s_p^{(-)}} - [M_{ns}]_{s_p^{(+)}} \right\} = 0. \tag{15}$$

Since the boundary is considered traction-free, the natural choice for the third relation in (15) is

$$[M_{ns}]_{s_p^{(-)}} = [M_{ns}]_{s_p^{(+)}} = 0. \tag{16}$$

We shall refer to this effect in the second numerical example.

4. Numerical experiments

In this section we present the results of our numerical experiments using EBFs.

4.1. The analysis of very thin plates based on FSDT and TSDT

Several bending problems of isotropic and anisotropic laminated plates based on the first order and third order shear deformation theories with varying ratios of thickness-to-side have