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High-order free vibration analysis of sandwich beams with a flexible core using dynamic stiffness method

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ABSTRACT

In this paper, high-order free vibration of three-layered symmetric sandwich beam is investigated using dynamic stiffness method. The governing partial differential equations of motion for one element are derived using Hamilton's principle. This formulation leads to seven partial differential equations which are coupled in axial and bending deformations. For the harmonic motion, these equations are divided into two ordinary differential equations by considering the symmetrical sandwich beam. Closed form analytical solutions of these equations are determined. By applying the boundary conditions, the element dynamic stiffness matrix is developed. The element dynamic stiffness matrices are assembled and the boundary conditions of the beam are applied, so that the dynamic stiffness matrix of the beam is derived. Natural frequencies and mode shapes are computed by use of numerical techniques and the known Wittrick–Williams algorithm. Finally, some numerical examples are discussed using dynamic stiffness method.

1. Introduction

In recent years, the application of sandwich beams as primary members or structures with low weight and high strength and stiffness has widespread in various industries. The wide range and importance of these applications, stresses the need of accurate model capable of predicting the vibration response of sandwich structures [1–14].

In the most cases, the free vibration of sandwich beams with honeycomb cores has been studied using the incompressible core hypothesis [1–7]. Di Taranto [1] and Mead and Markus [2] are the earliest investigators who studied the free vibration of sandwich beams using the classical theory. Mead [3] made assess and compared the different models that are used to investigate the free vibration of sandwich beams. A simple model assumes that the top and the bottom face sheets of a sandwich beam deform according to the Bernoulli–Euler beam theory, whereas the core deforms only in shear. This model was used by many researchers. With the advent of digital computer, finite element based solutions are available [4]. In recent years, scientists have investigated free vibrations of sandwich beams using dynamic stiffness method [5–7]. There are many advantages of the dynamic stiffness method in that it is probably the most accurate method (often called as an exact method) and unlike the finite element and other approximate methods, the model accuracy is not unduly compromised, as a result of using a small number of elements in the analysis. Furthermore, one of the great advantages of the dynamic stiffness method is that the results are independent of the number of elements used in the analysis. For instance, one single structural element can be used to obtain any number of its natural frequencies and mode shapes to any desired accuracy. This is clearly impossible in the finite element and other approximate methods in which the results are generally, if not always, dependant on the number and quality of the elements used in the analysis. It is well known that the finite element and other approximate methods become more and more unreliable at higher frequencies.

This has in part motivated the current work, which sets out to derive the dynamic stiffness matrix to use it to investigate the free vibration characteristics of the sandwich beams.

Furthermore, Khalili et al. [7] showed the application of dynamic stiffness method to analyze the sandwich beam with the attachments and the elastic supports.

Widespread use of polymer foam materials as cores of sandwich structures requires application of an enhanced theory that accounts for the vertical flexibility of a soft foam core. Therefore, Frosting and Baruch [8] presented different model for sandwich beams analysis. They were analyzed the free vibrations of sandwich beams with flexible core based on the high-order theory. In this model, dynamic displacement of the core with variation of

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Nomenclature

$\mathbf{a_e}$	element constant vector
A_c	vertical area of core
A_f	vertical area of each face sheet
$A_j, \overline{A}_j$	jth constant indexes
b	width of the element
$B_j, \overline{B}_j$	jth constant indexes
$\overline{C}_j$	jth constant index
D	differential operator
E_c	Young's modulus of core
E_f	Young's modulus of each face sheet
$\mathbf{f_e}$	element force vector
G_c	shear's modulus of core
h_c	thickness of core
h_f	thickness of each face sheet
i	$\sqrt{-1}$
i	counter for natural numbers
I_f	second moment of area of each face sheet
j	counter for natural numbers
$\mathbf{K^{DSM}}$	overall dynamic stiffness matrix
$\mathbf{K_e^{DSM}}$	element dynamic stiffness matrix
l	length of an element
L	length of the beam
M_b, M_t	bending moment of face sheets
$M_{0b}, M_{0t}, M_{lb}, M_{lt}$	bending moment of nodes
P_b, P_t	axial force of face sheets
$P_{0b}, P_{0t}, P_{lb}, P_{lt}$	axial force of nodes
t	time
T	kinetic energy
$\mathbf{u}$	overall displacement vector
$\mathbf{u_e}$	element displacement vector
u_b, u_c, u_t	axial displacement of layers
$u_{0b}, u_{0t}, u_{lb}, u_{lt}$	axial displacement of nodes
U	strain energy
V_b, V_c, V_t	shear force of layers
$V_{0b}, V_{0c}, V_{0t}, V_{lb}, V_{lc}, V_{lt}$	shear force of nodes
$\mathcal{V}_b, \mathcal{V}_c, \mathcal{V}_t$	volume of layers
w_b, w_c, w_t	vertical displacement of layers
$w_{0b}, w_{0c}, w_{0t}, w_{lb}, w_{lc}, w_{lt}$	vertical displacement of nodes
x	member axis
X	axial direction
y	member axis
Y	vertical direction
Z	width direction
γ_{xy}	shear strain
δ	first variation operator
ε_x	normal strain
η	non-dimensional variable
$\lambda_j, \bar{\lambda}_j$	jth roots of characteristic equation
θ_b, θ_t	rotation displacement of face sheets
$\theta_{0b}, \theta_{0t}, \theta_{lb}, \theta_{lt}$	rotation displacement of nodes
ρ_c	density of core
ρ_f	density of each face sheet
σ_x	normal stress
τ_{xy}	shear stress
ω	natural frequency
$\Phi, \overline{\Phi}$	displacement functions
$(\dot{\cdot})$	$\frac{d}{dt}$
$(\ddot{\cdot})$	$\frac{d^2}{dt^2}$
$(')$	$\frac{d}{dx}$
$('')$	$\frac{d^2}{dx^2}$
$(''')$	$\frac{d^3}{dx^3}$
$('''')$	$\frac{d^4}{dx^4}$

thickness was assumed as linear polynomial. Finally, the axial and the transverse displacements of the core were obtained with two and three order polynomials, respectively [8,9]. Many authors are investigated on the high-order free vibration of sandwich beam [8–14].

Sokolinsky et al. [9] used this dynamic analysis formulation to examine the influence of boundary conditions on the free vibrations of sandwich beams. Finite differences were used to approximate the governing equations, and the deflated iterative Arnoldi algorithm was applied to solve the algebraic eigenvalue problem.

Sokolinsky and Nutt [10] provided a method to achieve a consistent formulation by using the compatibility relation between core and one of the face sheets. Sokolinsky et al. [11] predicated the natural frequencies and corresponding vibration modes of a cantilever sandwich beam with a soft polymer core using the higher-order theory by the two-dimensional finite element analysis and the experimental measurements. The comparison of their results with experimental results and those obtained using the classical formulation of Mead and Markus [2] showed the superiority of the high-order formulation.

Yang and Qiao [12] presented a higher-order impact model to simulate the response of a soft-core sandwich beam subjected to a foreign object impact. Before the impact analysis, they presented three higher-order models of sandwich beam that the dynamic effect of the core was different in them. In first model (A), they assumed that core was designed so light in comparison with the face sheets that the mass inertia of the core material could be omitted. In model (B), the horizontal vibration and rotatory inertia of the core and the face sheets were considered by using the assumption that the acceleration of the core can be approximated by a linear interpolation of the top face sheet and the bottom face sheet accelerations. In third model (C), the full dynamic effect (i.e., besides the mass inertia of the core, both the horizontal vibration and rotatory inertia of the core are also included) was considered by using the linear interpolation of the top face sheet and the bottom face sheet accelerations. Then, the free vibration problem of the sandwich beams was solved, and the results were validated by comparing with numerical finite element modeling results of ABAQUS.

Bekuit et al. [13] presented a quasi-two-dimensional finite element formulation for the static and dynamic analysis of sandwich beams. The through-the-thickness variation of each displacement field in each layer was expanded in polynomials and the span-wise variation was interpolated by the use of Lagrange cubic shape functions. Arvin et al. [14] presented the higher order theory for analysis of sandwich beam with composite faces and viscoelastic core by using finite element method. In addition, the effects of Young modulus, rotational inertia and core kinetic energy are considered to modify the Mead and Markus [2] theory.

In the present paper, the high-order free vibrations of sandwich beams are carried out using the dynamic stiffness method. First, the governing partial differential equations of motion for one element are derived using Hamilton's principle. For the harmonic motion, these equations are divided into two ordinary differential equations by considering the symmetrical sandwich beam, which apply to both axial and bending deformations. In other words,