Composite Structures 94 (2012) 1605–1615

Contents lists available at SciVerse ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Buckling of orthotropic micro/nanoscale plates under linearly varying in-plane load via nonlocal continuum mechanics

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ARTICLE INFO

Article history:
Available online 8 January 2012

This article is lovingly dedicated to the memory of Mr. Mohammad Farajpour, Ali Farajpour's uncle. He was always a source of motivation and inspiration.

Keywords:
Orthotropic nanoplate
Buckling
Nonlocal elasticity theory
Differential quadrature method (DQM)
Power series method (PSM)

ABSTRACT

Buckling response of orthotropic single layered graphene sheet (SLGS) is investigated using the nonlocal elasticity theory. Two opposite edges of the plate are subjected to linearly varying normal stresses. Small scale effects are taken into consideration. The nonlocal theory of Eringen and the equilibrium equations of a rectangular plate are employed to derive the governing equations. Differential quadrature method (DQM) has been used to solve the governing equations for various boundary conditions. To verify the accuracy of the present results, a power series (PS) solution is also developed. DQM results are successfully verified with those of the PS method. It is shown that the nonlocal effects play a prominent role in the stability behavior of orthotropic nanoplates. Furthermore, for the case of pure in-plane bending, the nonlocal effects are relatively more than other cases (other load factors) and the difference in the effect of small scale between this case and other cases is significant even for larger lengths.

1. Introduction

Invention of carbon nanotubes (CNTs) initiated a new era in the nano world [1]. Since then, many works have been performed in the field of mechanical, electrical, physical and chemical behaviors of nanostructures. Primary studies showed that the mechanical properties of nanostructures are different from other well-known materials [2]. The superior properties of these structures have led to its applications in many fields such as nanodevices, nanosensors, nanooscillators, nanoactuators, nanobearings, hydrogen storage, electrical batteries, and nanocomposites [3–5]. Continuum modeling of nanomaterials has received the great deal of attention of scientific community because controlled experiments in nanoscale are difficult and molecular dynamic simulations are highly computationally expensive. There are various size-dependent continuum theories such as couple stress theory [6], strain gradient elasticity theory [7], modified couple stress theory [8] and nonlocal elasticity theory [9–11]. Among these theories, nonlocal elasticity theory has been widely applied [12–22]. To overcome the shortcomings of classical elasticity theory, Eringen and Edelen [9] introduced the nonlocal elasticity theory in 1972. He modified the classical continuum mechanics for taking into account the small scale effects. According to the nonlocal elasticity theory, the stress tensor at an arbitrary point in the domain of nanomaterial depends not only on the strain tensor at that point but also on strain tensor at all other points in the domain. Both atomistic simulation results and experimental observations on phonon dispersion have shown the accuracy of this observation [10,23].

Peddieson et al. [12] first used the nonlocal elasticity theory to develop a nonlocal Benoulli/Euler beam model. After that, many researchers have employed the nonlocal elasticity theory for the investigation of vibration and buckling behaviors of nanostructures. Such nanostructures include nanotubes [13–19], nanorods [20], nanorings [21] and nanoplates [22]. Mechanical characteristics of carbon nanotubes (CNTs) have been investigated more than other types of nanomaterials [13–19]. Recently, the other nanostructures such as nanoplates have attracted the attention of scientific community. Most of the studies on mechanical properties of nanoplates have been carried out on graphene sheets. Stankovich et al. [24] developed a process for obtaining single layered graphene sheets (SLGSs) from graphite. The graphene sheets are widely used in the micro electro-mechanical systems (MEMS) and nano electro-mechanical systems (NEMS) [3,4]. The applications of graphene sheets in electro-mechanical resonators are studied by Bunch et al. [25]. Sakhaee-Pour et al. [26] explored the possibility of applying the SLGSs as mass sensors and atomistic dust detectors. They also used molecular structural mechanics to investigate the vibration characteristics of defect-free SLGSs, which have potential applications as strain sensors [27]. Furthermore, superior mechanical, thermal and electrical properties of graphene sheets make them favorable for creating novel composite materials with desirable physical characteristics [28]. Graphene-based nanocomposites as a new class of composite materials have attracted the attention

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doi:10.1016/j.compstruct.2011.12.032

of scientific community due to their wide potential applications in nanoengineering devices. Recently, several graphene-based nanocomposites have been successfully synthesized. It has been revealed that adding graphene sheets to polymer matrix could improve the mechanical properties greatly [29]. Raghu et al. [30] prepared a nanocomposite of waterborne polyurethane with graphene sheets that are a new type of nano-sized conductive filler. Furthermore, bio-nanocomposites films are in the early state of development that hold promise for many applications in biosensing and biotechnology [31,32]. In addition, the applications of the graphene-based nanocomposites for fabricating supercapacitors [33] and as anode material for lithium-ion batteries [34] have been reported. Due to these applications, understanding the mechanical behaviors of graphene sheets such as buckling is essential for their engineering design and manufacture.

Nanoscale vibration analysis of multi-layered graphene sheets (MLGSs) embedded in an elastic medium has been studied by Behfar and Naghdabadi [35]. Also, Liew et al. [36] have proposed a continuum-based plate model for the vibration behavior of multi-layered graphene sheets (MLGSs) that are embedded in an elastic matrix. Duan and Wang [37] obtained an exact closed form solution for the axisymmetric bending of micro- and nano-scale circular plates based on the nonlocal continuum mechanics. Pradhan and Murmu [38,39] used the nonlocal elasticity theory and differential quadrature method (DQM) for the buckling analysis of rectangular single layered graphene sheets under biaxial compression with and without the surrounding elastic medium. Further, they investigated the stability of biaxially compressed orthotropic plates at small scales [40] because it has been reported that the graphene sheets have orthotropic properties [41]. Aghababaei and Reddy [42] reformulated the third-order shear deformation plate theory for the vibration and bending of nanoplates. In addition, the higher order shear deformation theory (HSDT) was developed for the buckling of single-layered graphene sheets [43]. Using the nonlocal finite element model, Ansari et al. [44] determined the natural frequencies of multi-layered graphene sheets. Wang et al. [45] investigated the small scale effects on the longitudinal wave propagation in nanoplates. Based on nonlocal continuum theory, they also explored the flexural wave propagation in nanoplate embedded in elastic matrix with initial stress [46]. Babaei and Shahidi [47] reported the buckling behaviors of various quadrilateral nanoplates such as skew, rhombic and trapezoidal nanoplates. The free vibration of circular nanoplates with consideration of surface properties was investigated by Assadi and Farshi [48]. In another work, an exact solution for three dimensional vibration analysis of nanoplates by decoupling the field equations of Eringen theory, has been reported [49]. The Levy type method and nonlocal plate model have also been used in the vibration and buckling analyses of nanoplates [50]. Furthermore, Malekzadeh et al. [51,52] investigated the small scale effect on the thermal buckling and vibration of orthotropic arbitrary straight-sided quadrilateral nanoplates with the nonlocal elasticity theory.

In the present work attempt is made to study the buckling characteristics of orthotropic graphene sheets under various linearly varying in-plane normal forces. Based on the nonlocal elasticity theory, the small scale effects are introduced. Using the equilibrium equations of a differential element of a rectangular plate, the governing equations of single layered graphene sheet (SLGS) are derived. Differential quadrature method (DQM) is used to solve the governing equations for simply supported boundary conditions, clamped boundary conditions and various combinations of them. To verify the accuracy of the DQM solutions, the governing equation is also solved by the power series method (PSM) of Frobenius. The predicted results by the DQ technique are successfully verified with those of the PSM solution. The small scale effects on the bucking loads of graphene sheets are investigated through considering various parameters such as numerical loading factor, the length of nanoplate, nonlocal parameter, mode number and aspect ratio. It is anticipated that the results of the present work would be helpful for designing NEMS/MEMS components using single layered graphene sheets.

2. Formulation

Consider an orthotropic rectangular nanoplate with principal directions parallel to the sides of the plate. Cartesian coordinate frame with axes x, y and z, continuum plate model and discrete model used for the single layered graphene sheet (SLGS) are shown in Fig. 1. The origin of the coordinate system is placed at the lower left corner of the midplane of the plate. The x and y axes are chosen along the length and width of nanoplate, respectively. The dimensions of the nanoplate are l_x (the length of the plate) and l_y (the width of the plate).

The changes that are caused by the decrease in the size of a body at small scale are called size effects or nonlocal effects. Both experimental studies and molecular dynamics (MD) simulations have shown that the small scale effects (size effects) play an important role in the mechanical properties of nanostructures [10,23]. As the dimensions of these structures are reduced, the effects of intermolecular and inter-atomic interactions should be considered to predict mechanical behavior properly. The classical elasticity theory does not involve these effects. Therefore, it is required to modify the classical theory to include small scale effects. For this purpose, Eringen and co-workers [9–11] captured the small scale effects by assuming the stress at a point as a function not only of the strain at that point but also a function of the strains at all other points in the domain. Nonlocal theory considers long-range interatomic interaction and yields results dependent on the size of a body. The classical elasticity theory is a special case of the nonlocal theory in which stress state at an arbitrary point depends only on the strain state at that point. According to nonlocal elasticity theory [9–11], the basic stress–strain equation for a Hookean solid neglecting the body force is expressed by the following partial-integral constitutive relationship:

$$\sigma_{ij}^{nl} = \int\int_V\int \varphi(|x - x'|, \gamma)\sigma_{ij}^{l} dV \tag{1}$$

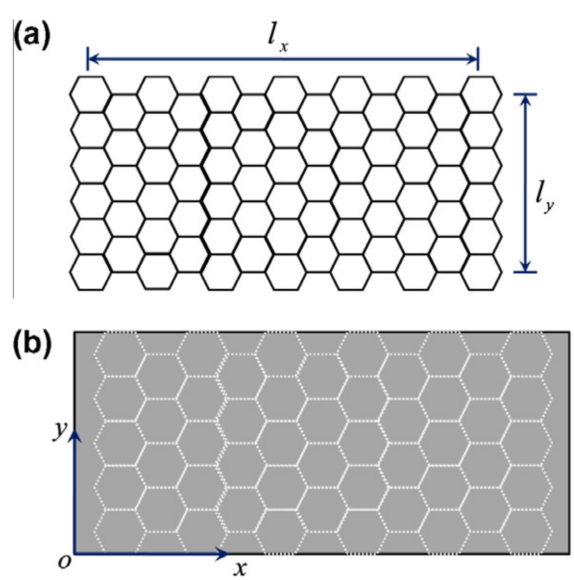

Fig. 1. Models of rectangular single layered graphene sheet. (a) Discrete model and (b) continuum model.