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Continuum damage mechanics approach to composite laminated shallow cylindrical/conical panels under static loading

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ABSTRACT

Static response characteristics and failure load of laminated composite shallow cylindrical and conical panels subjected to internal/external lateral pressure are investigated using continuum damage mechanics approach considering geometric nonlinearity and damage evolution. The damage model is based on a generalized macroscopic continuum theory within the framework of irreversible thermodynamics and enables to predict the progressive damage and failure load. Damage variables are introduced for the phenomenological treatment of the state of defects and its implications on the degradation of the stiffness properties. The analysis is carried out using finite element method based on the first order shear deformation theory. The nonlinear governing equations are solved using Newton–Raphson iterative technique coupled with the adaptive displacement control method to efficiently trace the equilibrium path. The detailed parametric study is carried out to investigate the influences of geometric nonlinearity, evolving damage, span-to-thickness ratio, lamination scheme and semi-cone angle on the static response and failure load of laminated cylindrical/conical panels. It is revealed that the membrane forces due to geometric nonlinearity significantly influence the damage distribution and failure load.

1. Introduction

Microscopic damage of fibre reinforced laminated composites in the form of fibre breakage, fibre buckling, matrix cracking, fibre–matrix debonding and delamination can be quantified by representing the effects of distributed defects including material stiffness degradation in terms of internal state variables within the framework of continuum damage mechanics (CDM). Such an internal state variable was first introduced by Kachanov [1] to describe inelastic constitutive equations for isotropic solids. Utilizing this concept of damage variable, Robotnov [2] introduced the concept of effective stress. Lemaitre [3–5] and Sidoroff [6] gave the physical interpretation of damage variable by proposing the hypothesis of strain equivalence and strain energy equivalence, respectively. Chaboche [7,8] described different measures of the damage variable, developed damage growth equations within the thermodynamic general framework and also discussed the use of CDM for local approaches of fracture.

Based on strain equivalence hypothesis, Ladeveze and Dantec [9] formulated mesomechanical damage model for single-ply laminate considering matrix microcracking and fibre/matrix debonding represented by two internal state variables. The damage evolution law is assumed to be linear function of equivalent damage energy release rate. Allix and Ladeveze [10,11], Daudeville and Ladeveze [12], extended the approach [9] to predict the interface delamination by introducing an interface layer between the laminae and three damage variables corresponding to degradation of three interface stiffness constants. These damage variables are assumed to evolve according to a power law function of equivalent damage energy release rate. Ladeveze [13] and Ladeveze et al. [14] combined the ply and interface damage models to predict the overall damage of laminate under quasi-static and dynamic loadings.

Kennedy and Nahan [15,16] modelled damage at laminate level with the degradation of laminate stiffness coefficients. The evolution of damage variables is based on exponential function of nonlocal strains. Further, they extended this work to include the linear variation of orthotropic damage variables through the thickness [17]. Germain et al. [18] proposed anisotropic nonlocal damage model by introducing different internal lengths for each principal material direction. Three damage variables were expressed as exponential functions of nonlocal damage driving forces.

Matzenmiller et al. [19] presented failure mode-dependent anisotropic damage model of fibre-reinforced composites based on in-plane failure modes *i.e.* fibre failure, matrix failure, and fibre–matrix shear failure. The constitutive relation is based on strain equivalence principle and the damage variables are evaluated using the concept of thermodynamics of irreversible processes. Loading surface is derived from plane stress version of Hashin criterion in terms of damage energy release rate. The rate

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of change of damage variables is expressed as functions of damage growth and strain rate. Barbero and De Vivo [20], Barbero and Lonetti [21,22], and Lonetti et al. [23] coupled the concepts of continuum damage mechanics with thermodynamics of irreversible processes and proposed a model for damage initiation, evolution, and failure at critical values of damage for polymer matrix composites.

The implementation of continuum damage mechanics model [20] into a finite element model based on the first order and layer-wise shear deformation theories for fibre-reinforced composite laminated plate is presented by Robbins et al. [24,25]. Ghosh and Sinha [26] and Ghosh [27] have studied impact response of laminated composite plates and spherical panels using damage variable evolution equation for uniaxial loading [19] to evaluate damage variables in the principal material directions. In the studies [24–27], geometric nonlinearity was not considered. The nonlinear dynamic response of laminated plates with piezoelectric layers is studied by Tian et al. [28,29] considering elasto-plastic deformation, damage evolution law similar to plastic flow rule and classical plate theory. The elasto-plastic postbuckling of orthotropic plates is studied by Tian and Fu [30] using classical plate theory and continuum damage based on damage flow rule. Tian et al. [28,29] and Tian and Fu [30] have used stress tensor as the conjugate force to damage which is thermodynamically inconsistent [31]. Fu et al. [32] investigated the effect of damage on elasto-plastic impact response of laminated composite shallow spherical shell under low velocity impact using first order shear deformation theory, axisymmetric model and damage initiation/propagation based on Hashin failure criterion.

It can be concluded from the literature review that most of the studies are mainly concerned with development of damage model. The implementation of these models for analysis of composite laminated cylindrical and conical panels has not received the attention of researchers. For the thin structural elements undergoing transverse displacements of the order of their thickness, inclusion of geometric nonlinearity is important. The effect of geometric nonlinearity coupled with continuum damage mechanics on the load carrying capacity of laminated cylindrical and conical panels has not been investigated.

In the present work, behaviour of composite laminated shallow panels is studied incorporating continuum damage mechanics and geometric nonlinearity. The study is carried out using finite element method based on first order shear deformation theory. A detailed parametric study is carried out to investigate the effects of damage, geometric nonlinearity, span-to-thickness ratio, boundary conditions, radius-to-span ratio, lamination scheme, and semi-cone angle on the static response characteristics of laminated shallow panels subjected to uniformly distributed transverse load.

2. Continuum damage mechanics model

2.1. Damage constitutive equations

The relation between the Cauchy stress $\{\boldsymbol{\sigma}\}$ in the damaged continuum and effective stress $\{\bar{\boldsymbol{\sigma}}\}$ in the equivalent undamaged continuum can be expressed as [33]:

$$\{\bar{\boldsymbol{\sigma}}\} = [\mathbf{M}]\{\boldsymbol{\sigma}\} \tag{1}$$

where $[\mathbf{M}]$ is the effective damage tensor. The non-zero elements of the effective damage tensor [34] are given by

$$\begin{aligned} M_{11} &= 1/(1-D_1), \quad M_{22} = 1/(1-D_2), \quad M_{33} \\ &= 1/(1-D_3), \quad M_{44} = \sqrt{1/(1-D_1)(1-D_2)}, \quad M_{55} \\ &= \sqrt{1/(1-D_1)(1-D_3)}, \quad M_{66} = \sqrt{1/(1-D_2)(1-D_3)} \end{aligned} \tag{2}$$

where D_1, D_2, D_3 are the damage variables along the principal material directions. The constitutive relations for damaged and equivalent undamaged configurations are given as:

$$\{\boldsymbol{\sigma}\} = [\bar{\mathbf{C}}^{\mathbf{e}}]\{\boldsymbol{\varepsilon}\}, \quad \text{and } \{\bar{\boldsymbol{\sigma}}\} = [\mathbf{C}^{\mathbf{e}}]\{\bar{\boldsymbol{\varepsilon}}\} \tag{3}$$

where $[\bar{\mathbf{C}}^{\mathbf{e}}]$ and $[\mathbf{C}^{\mathbf{e}}]$ are constitutive matrices of damaged and undamaged continua, respectively, $\{\boldsymbol{\varepsilon}\}$ and $\{\bar{\boldsymbol{\varepsilon}}\}$ are respective strains. The principle of equivalence of strain energy in the damaged and undamaged configuration is given by [6]:

$$\frac{1}{2}\{\boldsymbol{\sigma}\}^{\mathrm{T}}[\bar{\mathbf{C}}^{\mathbf{e}}]^{-1}\{\boldsymbol{\sigma}\} = \frac{1}{2}\{\bar{\boldsymbol{\sigma}}\}^{\mathrm{T}}[\mathbf{C}^{\mathbf{e}}]^{-1}\{\bar{\boldsymbol{\sigma}}\} \tag{4}$$

Substituting Eq. (1) in Eq. (4), the constitutive matrix of damaged state can be expressed in terms of the principal damage variables and constitutive matrix of undamaged state as:

$$[\bar{\mathbf{C}}^{\mathbf{e}}] = [\mathbf{M}]^{-1}[\mathbf{C}^{\mathbf{e}}][\mathbf{M}]^{-\mathrm{T}} \tag{5}$$

2.2. Damage evolution equations

The damage evolution equations are derived using principles of irreversible thermodynamics. The Clausius–Duhem inequality is expressed as [5]:

$$\left(\{\boldsymbol{\sigma}\} - \rho\frac{\partial\Psi}{\partial\{\boldsymbol{\varepsilon}\}}\right)^{\mathrm{T}}\{\dot{\boldsymbol{\varepsilon}}\} - \left(\rho\frac{\partial\Psi}{\partial\{\mathbf{D}\}}\right)^{\mathrm{T}}\{\dot{\mathbf{D}}\} - \rho\frac{\partial\Psi}{\partial\beta}\dot{\beta} \geqslant 0 \tag{6}$$

where ρ, ψ and β are material density, Helmholtz free energy and overall damage parameter; $\{\dot{\boldsymbol{\varepsilon}}\}$, $\{\dot{\mathbf{D}}\}$ and $\dot{\beta}$ are time derivatives of strain, damage and overall damage parameter, respectively. The thermodynamic damage driving force $\mathbf{Y}$ and damage hardening variable γ are defined as:

$$\{\mathbf{Y}\} = -\rho\frac{\partial\Psi}{\partial\{\mathbf{D}\}} = -\frac{\partial(\frac{1}{2}\{\boldsymbol{\varepsilon}\}^{\mathrm{T}}[\bar{\mathbf{C}}^{\mathbf{e}}]\{\boldsymbol{\varepsilon}\})}{\partial\{\mathbf{D}\}} = -\frac{1}{2}\{\boldsymbol{\varepsilon}\}^{\mathrm{T}}\frac{\partial[\bar{\mathbf{C}}^{\mathbf{e}}]}{\partial\{\mathbf{D}\}}\{\boldsymbol{\varepsilon}\} \tag{7a}$$

$$\gamma = \gamma(\beta) = -\rho\frac{\partial\Psi}{\partial\beta} \tag{7b}$$

In order to predict the evolution of damage $\mathbf{D}$, a convex damage surface is considered [20]:

$$g(\mathbf{Y},\gamma) = \sqrt{J_{11}Y_1^2 + J_{22}Y_2^2 + J_{33}Y_3^2} - (\gamma_0 + \gamma(\beta)) \tag{8}$$

where γ_0 is a material constant representing the initial damage threshold of undamaged material, $\gamma(\beta)$ provides the increase in damage hardening with evolution of damage variables and J_{11}, J_{22}, J_{33} are damage tolerance parameters of the material.

The damage state variables D_i and β are determined such that the dissipation function defined by $\phi(\mathbf{Y},\gamma) = Y_i\dot{D}_i - \gamma\dot{\beta} \geqslant 0$ is maximum subject to the constraint $g(\mathbf{Y},\gamma) = 0$. Using Lagrange multiplier approach, evolution equations for $\dot{D}_i$ and $\dot{\beta}$ are obtained as:

$$\dot{D}_i = \lambda\frac{\partial g}{\partial Y_i}, \quad \text{and } \dot{\beta} = \lambda\frac{\partial g}{\partial\gamma} \tag{9}$$

where $\lambda = (\frac{\partial g}{\partial Y_i} / \frac{\partial\gamma}{\partial\beta})\dot{Y}_i$ and $\dot{Y}_i$ is the time derivates of damage driving force Y_i.

Rate-type equations of damage evolution are expressed in the form of incremental thermodynamic damage driving force $\mathbf{Y}$ and damage hardening variable γ which are functions of incremental strain vectors and overall damage parameter, respectively. In the finite element formulation, damage evolution equations are solved at every Gauss point using Newton–Raphson iterative technique. A linear relationship $\gamma = k\beta$ is used to determine the damage hardening variable γ representing the damage hardening characteristics of composite laminates.