Composite Structures 94 (2012) 1793–1804

Contents lists available at SciVerse ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Interlaminar stresses in thick rectangular laminated plates with arbitrary laminations and boundary conditions under transverse loads

M. Tahani *, A. Andakhshideh

Department of Mechanical Engineering, Faculty of Engineering, Ferdowsi University of Mashhad, Mashhad, Iran

ARTICLE INFO

Article history:
Available online 5 January 2012

Keywords:
Composite laminated plates
Edge effect
Interlaminar stresses
Three-dimensional multi-term extended Kantorovich method
Transverse loading

ABSTRACT

In the present study, interlaminar stresses resulting from bending of thick rectangular laminated plates with arbitrary laminations and boundary conditions are analyzed analytically based on a three-dimensional multi-term extended Kantorovich method (3DMTEKM). Using the principle of minimum total potential energy, three systems of coupled ordinary differential equations with non-homogeneous boundary conditions are obtained. Then an iterative procedure is established to achieve analytical solution. The results obtained from this theory are compared with those of analytical solutions existing in the literature. It is found that the present results have excellent agreements with those obtained by layerwise theory. The results show that the multi-term EKM converges within only three terms of trial functions and the single-term EKM is not able to estimate the local interlaminar stresses near the boundaries of laminates. Finally, the power of the present approach in obtaining the interlaminar stresses in thick rectangular laminated plates with general types of boundary conditions and lay-ups is examined.

1. Introduction

Interlaminar stresses which arise at the edges of laminated composites play an important role in their analysis and design. They are a major source of concern in composite laminates because they can lead to delamination and failure of laminate at a much lower load than that predicted by in-plane failure criteria.

Numerous investigators have proposed various analytical and numerical methods to examine the transverse stress behavior near the edges of laminates. However, because of inherent complexities involved in the problem, no exact solution is known for elasticity equations. Therefore, various approximate methods for determining the interlaminar stresses are documented in the literature. These methods may, for convenience, be classified as either analytical or numerical. Because of the exceptionally large number of papers on the subject matter, only the pertinent pioneering works are referred here. The interested reader will find sufficient references to cover the literature in more depth in the review article by Kant and Swaminathan [1].

The first approximate solution of interlaminar shear stresses was proposed by Puppo and Evenson [2] based on a laminated model containing anisotropic layers separated by isotropic adhesive layers with interlaminar normal stress being neglected through the laminate. They showed that interlaminar shear stresses reach their maximum magnitudes at the free edges. Pipes and Pagano [3] subsequently obtained interlaminar stresses in a long symmetric laminate subjected to uniform axial strain. They utilized classical linear theory of elasticity to reveal the presence of significant stresses, both normal and shear, at the edges, between plies of composite plates. They employed a finite difference technique to solve the coupled second-order partial differential governing equations in terms of displacements. Whitney [4] also evaluated free edge stresses in laminated composites using simple stress approximations in the form of products of exponential and trigonometric functions. Later, Pagano [5] examined the problem by extension of the higher-order plate theory. Tang [6] and Tang and Levy [7] presented an analytical method by means of a boundary-layer theory. In this case they were able to predict all three interlaminar stress components near free edges of a symmetric angle-ply composite laminate under uniaxial tension. Using perturbation techniques, Hsu and Herakovich [8] developed a zeroth-order solution for investigation of edge effects in symmetric angle-ply laminates. Wang and Choi [9,10] studied the free edge singularities by means of Lekhnitskii's stress potential and the theory of anisotropic elasticity.

Reissner [11–13] proposed a valuable mixed variational theorem for the approximate analysis of isotropic and anisotropic laminated elastic plates and shells. Reissner's mixed variational theorem (RMVT) is a better description than classical variational formulation with only displacement variables. The classical variational theories do not provide a priori interlaminar continuous

* Corresponding author. Tel./fax: +98 511 876 3304.
E-mail address: mtahani@ferdowsi.um.ac.ir (M. Tahani).

0263-8223/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compstruct.2011.12.027

transverse stresses. Therefore, a post-processing procedure is required to recover interlaminar stresses. However, RMVT enforces compatibility of transverse strains by means of Lagrange multipliers which are variation of transverse stresses. To this end, the transverse normal and shear stresses as well as displacements will be continuous through the thickness of multilayered structures. Moreover, Carrera [14] presented the use of RMVT for two-dimensional modeling of flat and curved multilayer structures. He introduced various ways in which RMVT can be used to develop plate and shell theories in a systematic manner. He also examined the performance of RMVT by comparing its numerical results with those of exact elasticity solutions and with other significant analyses based on classical and refined approaches. He concluded that RMVT should be considered as a natural tool for analyses of multilayered structures.

Recently, Nosier and Bahrami [15,16] analyzed the edge effect problem of an antisymmetric angle-ply laminate subjected to arbitrary combinations of extensional and torsional loads. They developed analytical solutions to the edge-effect problem of such laminates under a uniform axial strain using the first-order shear deformation theory of plates and Reddy's layerwise theory. Nosier and Maleki [17] developed an analytical method in order to accurately calculate the interlaminar stresses near the free edges of generally long flat laminated composite plates under extension. They verified the accuracy and effectiveness of the proposed improved first-order shear deformation theory by means of comparison with the results of Reddy's layerwise theory as a three-dimensional benchmark.

Among the studies of this subject, one can find several papers in the field of interlaminar stresses near laminate's edges under pure bending [18] or general types of lateral loads [19–23]. Tang [19] developed an approximate one-term boundary layer solution for a uniformly loaded rectangular plate in bending. In this paper, interlaminar stresses in symmetric angle-ply composite laminates with two sides simply supported and the other two sides free was investigated. A global high-order shear deformation theory was developed by Lo et al. [20] to study cylindrical bending of angle-ply laminates. They investigated simply supported symmetric laminates subjected to a pressure on the top surface of the plates. Salamon [21] predicted the interlaminar stresses in four-layer $[\pm 45]_s$ and $[0/\theta]_s$ laminates under uniform bending using a finite difference approach to solve the exact elasticity equations. Murthy and Chamis [22] studied the free edge stress fields in composite laminates using a three-dimensional finite element analysis. In their studies they considered composite laminates under different loading conditions including in-plane and out-of-plane bending, combined axial tension and in-plane shear, twisting, uniform temperature, and uniform moisture. Analytical solutions for displacements and stresses in simply supported composite laminates were developed using the laminate plate theory of Reddy by Barbero et al. [23]. They analyzed constant thickness laminates and neglected the transverse normal stress component. Their results indicated that the generalized shear deformation plate theory predicts accurate stress distributions in thick composite laminates.

Tahani and Nosier [24] determined analytically interlaminar stresses resulting from bending of rectangular cross-ply composite laminates using a layerwise laminate theory. They analyzed laminates with two edges simply supported and the other two edges free.

As already mentioned, because of inherent complexities involved in the governing equations of the edge-effect problem, approximate methods which are based on numerical or analytical approaches have been pursued. Although accurate numerical methods such as finite element method or finite difference method exist, analytical methods are preferred in most cases because of their high accuracy. Among analytical methods, one can refer to the extended Kantorovich method (EKM). The EKM [25] is an iterative method for the solution of partial differential equations (PDFs), which was originally implemented for problems consisting of homogeneous boundary conditions [26]. In the EKM, solution does not depend on the initial guess of functions and thus possesses a significant advantage when solving for any types of boundary conditions.

Cho and Yoon [27], using the EKM and the complementary minimum energy principle, estimated the interlaminar stresses at the straight free edges of laminates under extension loads. Later Cho and Kim [28] analyzed free edge interlaminar stresses of composite laminates under extension, bending, twisting, and thermal loads. In their studies the stresses, which satisfy the traction-free conditions not only at the free edges but also at the top and bottom surfaces of laminates, were obtained by using the complementary virtual work and the extended Kantorovich method. Because the stress functions satisfy the stress and displacement continuity conditions at the ply interfaces by iterations, the EKM provides good predictions for interlaminar stresses in the general loading cases. Kim et al. [29] proposed a stress function-based analysis to provide a simple and efficient approximation method for estimating three-dimensional state of stress near the free edge of bonded composite patches. Cho and Kim [28] and Kim et al. [29] assumed that a generalized plane strain state exists and solved only symmetric laminated composites.

Recently, Kapuria and Kumari [30] extended the EKM for three-dimensional (3D) elasticity problem of a transversely loaded laminated angle-ply flat panel in cylindrical bending. They presented application of the EKM in a cylindrical bending problem involving an in-plane direction and a thickness direction instead of both in-plane directions in 2D elasticity problems. It should be noted that a 2D single-term EKM was used is this paper to solve a plane strain problem.

It is well known that the bending of composite plates with general lay-up and arbitrary boundary conditions cannot be characterized by the single-term EKM [31]. In this case, the coupled material behavior of the laminated plate requires a more universal description for the unknown functions. Such description can be achieved through the multi-term expansion. The single-term EKM was found to yield good accuracy for displacements and stresses in the interior region of laminates. However, it did not accurately predict the local stress field at the vicinity of the boundaries of laminates.

In summary, it is pointed out here that most methods employed so far in the literature are neither general nor enough accurate. The analytical methods existing in the literature to study the edge-effect problem are limited to special cases, such as long laminates, Levy-type boundary conditions, and balanced symmetric lay-ups. In this paper an analytical method is developed, for the first time, based on three-dimensional multi-term extended Kantorovich method to calculate interlaminar stresses in thick rectangular composite laminated plates with arbitrary laminations and general boundary conditions. To the best of the author's knowledge, this approach is novel and can be developed for solving other 3D elasticity problems and the results in the edge-effect problem can be used as benchmark for future studies.

2. Theoretical formulation

A generally laminated composite plate subjected to transverse load as shown in Fig. 1 is considered here with a total thickness h, width b in the lateral (y-) direction, and length a in the longitudinal (x-) direction. The coordinate system (x, y, z) is located at the middle plane of the laminate.