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Free vibration analysis of functionally graded elliptical cylindrical shells using higher-order theory

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Abstract

Here, the free vibration characteristics of functionally graded elliptical cylindrical shells are analyzed using finite element formulated based on the theory with higher-order through the thickness approximations of both in-plane and transverse displacements. The power law variation of properties is assumed in the thickness direction. The finite element employed in the study is based on field-consistency approach and free from shear and membrane locking problems. The strain–displacement relations are accurately introduced in the formulation without making any approximation in the thickness co-ordinate to radius ratio terms. The detailed parametric studies are carried out to study the influences of non-circularity, radius-to-thickness ratio, material composition and material profile index on the free vibration frequencies and mode shape characteristics of functionally graded elliptical shells. The significance of thickness stretch/contraction terms is highlighted through the mode shape study.

Keywords: Functionally graded shell; Elliptical; Higher-order theory; Transverse normal strain; Free vibration

1. Introduction

The recent advancements made in material science and engineering, coupled with the ubiquitous necessity to create efficient structural systems, have led to the development of new class of materials called functionally graded materials (FGMs). These materials can be designed as thermal barrier or heat shields to withstand high temperatures and extremely large thermal gradients present in high-speed spacecraft, turbomachinary, and nuclear and chemical industries. The applications of FGMs can be extended to solid oxide fuel cells and energy conversion systems using thermoelectric or thermionic materials [1], tribology [2,3], and semiconductor devices [4]. The development of FGM in defence mechanisms such as ceramic armour, thermo-nuclear suits etc. is also an area of interest. FGMs are also found in nature e.g. the bamboo has macroscopic gradient structure along height and radial directions for adapting to bending stresses due to wind loading [5].

These materials are microscopically inhomogeneous but at macro level, the properties vary smoothly and continuously from one surface to the other. This characteristic is achieved by gradually varying the volume fraction of the constituent materials, usually in the thickness direction. Typically, these materials are made from a mixture of ceramic and metal or from a combination of different metals using powder metallurgy processes, slurry stacking and sintering process or chemical vapour deposition technique. Smooth transitions in compositional gradients lead to improved bi-material bonding, better mechanical integrity, reduced thermo-elastic property mismatch and reduction in interlaminar stresses. This eliminates the interface problems of laminated composite materials such as delamination, cracking and debonding present due to the abrupt change in material

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properties across an interface between discrete materials/layers. The properties of FGMs can be tailored by spatially varying the microstructure to meet the requirements of different applications and working environments. These novel materials were first introduced by a group of scientists in Sendai, Japan in 1984 [6] and then rapidly developed by other researchers [7,8].

The investigations on the static/dynamic characteristics of FGM shell structures due to their evident importance in practical applications has recently attracted the attention of many researchers. The important contributions are cited here. Loy et al. [9] investigated the free vibration of simply supported FGM cylindrical shells. This is extended to cylindrical shells with different support conditions in the work of Pradhan et al. [10]. The governing equations derived based on the classical shell theory assumption are solved using Rayleigh–Ritz method in Refs. [9,10]. The transient elastic response analysis of functionally graded cylindrical shells subjected to impact loading is studied in the literature using Reddy's third-order shear deformation theory without incorporating transverse normal deformation [11] and employing hybrid numerical method [12,13]. The dynamic instability characteristics of FGM cylindrical shells [14] using classical shell theory and cylindrical panels [15] using Reddy's third-order shear deformation theory are investigated using Bolotin's method.

Functionally graded piezoelectric shells subjected to static electro-mechanical loading situations are analyzed using linear through the thickness approximation of in-plane and transverse displacements and quadratic variation of electric potential in the work of Wu et al. [16]. Active control of FGM shells using piezoelectric sensors/actuators has been studied by Liew et al. [17] and He et al. [18] using the classical shell theory. Postbuckling analysis of functionally graded cylindrical shells/panels in thermal environment subjected to axial compressive force/external pressure has been carried out using classical theory [19,20] and Reddy's third-order theory [21,22]. Thermal buckling of functionally graded material cylindrical shells subjected to uniform, linear and non-linear temperature distribution through the thickness has been investigated by Shahsiah and Eslami [23,24]. In all the above-cited work pertaining to cylindrical shells, except Ref. [16], the transverse normal deformation is neglected. However, the exact three-dimensional solutions for thick functionally graded structures [25–27] show that the transverse normal deformation is significant due to the gradation of properties in the thickness direction. Furthermore, the variation of transverse normal displacement through the thickness is quadratic in nature [26,27].

While the circular cylinders are perhaps the most common, either due to the design consideration, for instance in submersibles, flight vehicles etc., or due to fabrication process, the cross-section of such shells may become non-circular. The presence of non-circularity in the shell cross section may adversely affect the dynamic characteristics of the shell [28]. Hence, the study of the dynamic characteristics of functionally graded non-circular cylinders assumes importance. To the authors' knowledge the work on the behavior of functionally graded non-circular cylinders is not yet available in the literature.

In the present work, the free vibration characteristics of functionally graded material elliptical cylindrical shells using finite element procedure are studied using the higher-order displacement model including variable transverse displacement through the thickness. The strain–displacement relations are accurately accounted for in the formulation. The validation of the present model is carried out considering the available analytical solutions for cylindrical shells. A detailed parametric study is carried out to highlight the influences of elliptical cross-sectional parameter, thickness ratio and material profile index on the free vibration frequencies and mode shapes of functionally graded elliptical shells.

2. Formulation

A non-circular cylindrical shell, functionally graded in the thickness direction, is considered with the co-ordinates x along the meridional direction, y along the circumferential direction and z along the thickness direction having origin at the middle-surface of the shell as shown in Fig. 1. Based on Taylor's series expansion method for deducing the two-dimensional formulation of a three-dimensional elasticity problem, the in-plane displacements u and v, and the transverse displacement w are assumed as

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\theta_x(x,y,t) + z^2\beta_x(x,y,t) \\ &\quad + z^3\phi_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) + z\theta_y(x,y,t) + z^2\beta_y(x,y,t) \\ &\quad + z^3\phi_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) + zw_1(x,y,t) + z^2\Gamma(x,y,t) \end{aligned} \tag{1}$$

Here, u_0, v_0, w_0 are the displacements of a generic point on the reference surface; θ_x, θ_y are the rotations of normal to the reference surface about the y and x axes, respectively; $w_1, \beta_x, \beta_y, \Gamma, \phi_x, \phi_y$ are the higher order terms in the Taylor's series expansions, defined at the reference surface.

The strains in terms of middle-surface deformation, rotations of normal, and higher order terms associated with displacements are as,

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_{\mathrm{bm}} \\ \varepsilon_{\mathrm{s}} \end{Bmatrix} \tag{2}$$

The vector $\{\varepsilon_{\mathrm{bm}}\}$ includes the bending and membrane terms of the strain components and vector $\{\varepsilon_{\mathrm{s}}\}$ contains