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Post-buckling of cross-ply laminated rectangular plates containing short random fibers

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Abstract

Post-buckling of cross-ply laminated composite plate containing randomly oriented short spatial fibers in each layer is obtained analytically, using fast converging double Chebyshev series. The mathematical formulation is based on first-order shear deformation theory and von-Karman non-linearity. The effective elastic properties of the composites are expressed analytically in terms of phase properties, orientation angles, volume fraction, and fiber shape. The effects of fiber orientation in the composites, fiber volume fraction, fiber aspect ratio, and plate span to thickness ratio on the buckling and post-buckling strength are studied. Numerical results for E-glass/Epoxy fiber reinforced laminates are presented for the different boundary conditions and the number of layers of the composite. The results indicate that complete random distribution of the fibers in the composites gives higher buckling and post-buckling strength.

Keywords: Post-buckling; Laminated plate; Cross-ply; Short fibers; Analytical; Chebyshev series

1. Introduction

Composite laminated plates/panels have gained popularity in many advanced engineering structures. Laminated composite plates/panels are the main load bearing structural elements in modern aerospace, naval and other high performance engineering applications. Plate buckling problems have been the interest of many researchers over the years and the post-buckling behavior of plates is one of the major design criteria for an efficient and optimal usage of materials for plates. In addition to this the placement of the fibers in the composite is one of the efficient ways to improve the performance of the composites under different environmental and loading conditions. An improved through-the-thickness stiffness/strength and more balanced properties of the composite are obtained using random spatial short fibers and they also facilitate in formulating more complex shapes. However, it is almost impossible to control the movement of the fibers in perfect alignment and hence a probabilistic study on fiber orientation is necessary. The most essential material properties of such a composite are elastic moduli which governs the static, dynamic and buckling behaviors of the composite laminates. The fiber distribution in lamina is represented by either a density function or a cumulative function which helps in computing the elastic constants of the composites effectively. Several approaches have been reported in the literature for predicting the effective properties, notably among them are due to Eshelby [1], Halpin et al. [2], Christensen and Malls [3], Mori and Tanaka [4], Taya and Chou [5], Beveniste [6] and others. Weng [7] concluded that the Mori–Tanaka method can be applied to obtain the elastic properties of identical shaped multiphase composites with inclusions. Analytical expressions based on Mori–Tanaka mean field theory are presented in the present paper for evaluation of effective moduli of composites.

Buckling and post-buckling behavior of the laminated composite plates have received considerable attention in the past. Some of the excellent monographs and reviews are presented by Turvey and Marshall [8] and Argyris and Tenek [9]. Equivalent single layer

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approaches using displacement based first-order or higher-order shear deformation theories have been widely used for buckling analysis of laminated plates. Reddy and Phan [10] presented analytical solutions for the plate buckling problem using higher-order shear deformation theory. Khdeir [11] obtained Levy type solutions for buckling of cross-ply laminated plates incorporating Reddy's [12] higher-order theory. Buckling loads of composite plates were obtained analytically by Doong et al. [13], Ren and Owen [14], Matsunaga [15], and Kant and Swaminathan [16]. From the literatures available, it is evident that considerable amount of effort has been devoted to study the buckling and post-buckling behavior of composite laminated plates with composites having long fibers and mainly using numerical techniques. The post-buckling of composites containing randomly oriented spatial short fibers based on an analytical approach is rather limited. With the advancement in the computing tool, it is necessitated to present an analytical solution to post-buckling behavior of composite plates containing short spatial random fibers, which could serve as benchmark solutions.

In the present study, the post-buckling behaviors of composite laminated rectangular plates with randomly oriented spatial short fibers are studied. An analytical methodology of solution based on fast converging finite double Chebyshev series is employed. The effects of different cases of fiber orientation, fiber aspect ratio, volume fraction of the fibers, plate span to thickness ratio, and boundary conditions on buckling and post-buckling strength are studied.

2. Mathematical formulation

Fig. 1 shows the composite laminated rectangular plate having n layers with each layer containing randomly oriented short fibers. Perfect bonding between the layers is assumed. The evaluation of the effective elastic properties and the governing equations of equilibrium of the laminated composite plate are presented in the following subsections.

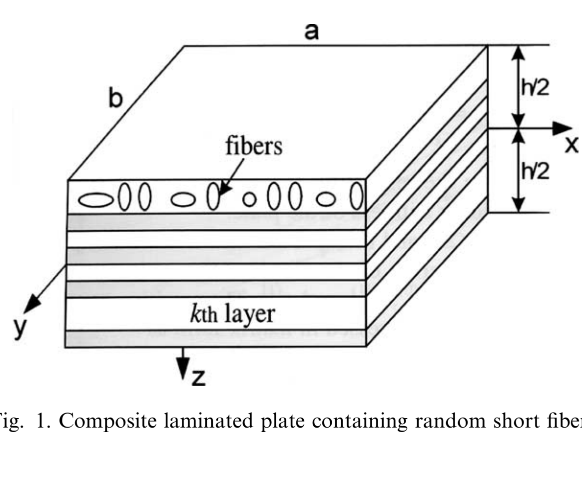

Fig. 1. Composite laminated plate containing random short fibers.

2.1. Effective elastic moduli

To simulate the fibrous geometry ranging from short fiber to continuous fiber, a spheroidal inclusion is defined by

$$\Omega:\ \frac{x^2}{a_1^2}+\frac{y^2}{a_2^2}+\frac{z^2}{a_3^2}\leqslant 1 \tag{1}$$

where $a_1(=a_2)$ and a_3 are the semi-axes of the spheroid as shown in Fig. 2. A composite consisting of randomly distributed spheroidal inclusions with elastic constants C^1_{ijmn} and volume fraction f is considered. The surrounding matrix is denoted by superscript 0 and has elastic constants C^0_{ijmn}. Using the Mori–Tanaka mean field theory [4] which accounts for interaction among inclusions, the effective elastic moduli C_{ijmn} for the two-phase composite are obtained explicitly by Huang and Kuo [17] as

$$C_{ijmn}=C^0_{ijmn}+fC^0_{ijab}T^{-1}_{abqr}(C^1_{qrmn}-C^0_{qrmn}) \tag{2}$$

where T^{-1}_{abij} is the inverse of T_{ijab} which is expressed as

$$T_{ijab}=C^0_{ijab}+(1-f)(C^1_{ijmn}-C^0_{ijmn})S_{mnab} \tag{3}$$

with S_{mnab} being the well known Eshelby Tensor [18].

The effective elastic moduli C_{ijmn} of the composite expressed in Eq. (2) is computed based on the assumption that the principal axes of fibers coincide with the directions of the composite matrix, but in practice it is difficult to achieve a perfect alignment. The effects of orientation on elastic properties of composites containing random oriented fibers are evaluated by taking the components of each tensor in Eq. (2) in the geometrical coordinates with reference to the global structure geometry. The orientation of a given spatially oriented fiber with respect to the global structure geometry is defined uniquely by three Euler angles θ, ϕ, and ω, as depicted in Fig. 3. For a misaligned fiber reinforced

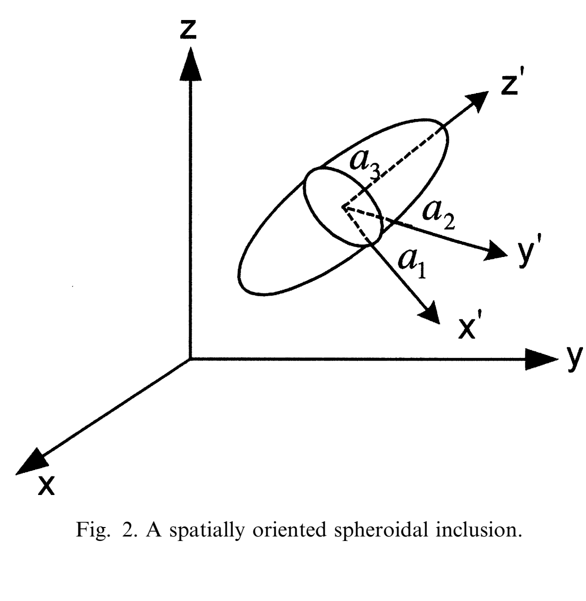

Fig. 2. A spatially oriented spheroidal inclusion.