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Vibration analysis of generally laminated composite plates by the moving least squares differential quadrature method

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Abstract

In this paper, a novel numerical solution technique, the moving least squares differential quadrature (MLSDQ) method is employed to study the free vibration problems of generally laminated composite plates based on the first order shear deformation theory. The weighting coefficients used in MLSDQ approximation are obtained through a fast computation of the MLS shape functions and their partial derivatives. By using this method, the governing differential equations are transformed into sets of linear homogeneous algebraic equations in terms of the displacement components at each discrete point. Boundary conditions are implemented through discrete grid points by constraining displacements, bending moments and rotations of the plate. Combining these algebraic equations yields a typical eigenvalue problem, which can be solved by a standard eigenvalue solver. Detailed formulations and implementations of this method are presented. Convergence and comparison studies are carried out to verify the reliability and accuracy of the numerical solutions. The applicability, efficiency, and simplicity of the present method are all demonstrated through solving several sample problems.

Keywords: Differential quadrature method; Moving least-squares method; Free vibration; Natural frequency; Laminated plate

1. Introduction

Composite laminated plates are extensively used in mechanical, civil, nuclear and aerospace structures due to their excellent advantages. Through proper arrangement of stacking sequence, fiber orientation, thickness and material properties of each layer, the strength and stiffness of the plate can be tailored to satisfying the given requirements. On the other hand, various coupling effects, such as stretching–bending, stretching–shearing, and bending–twisting couplings, etc., exist due to the anisotropy of the individual lamina and un-symmetric layering. The existence of these effects often diminishes the stiffness of the plate, and induces many complexities in analyzing the behavior of such plates. So it is very important to have a good understanding of the dynamic behavior of these structural components in the design and performance evaluation of mechanical systems.

A vast body of literature for vibration analysis of laminated plates is available. Analytical and numerical techniques for determining the vibration characteristics of laminated pates are abundant, well developed and widely studied. However, most of the previous studies are confined to the special cross-ply and angle-ply lamination (symmetric or anti-symmetric) with special boundary conditions. Few of them have been conducted on the arbitrarily laminated plates, in which case all kinds of couplings may exist, and correspondingly, the governing differential equations are highly coupled and hard to be solved. Baharlou and Leissa [1] presented an analysis of vibration and buckling of generally laminated plate having various boundary conditions using Ritz method, based on the classical plate theory. Darvizeh et al. [2] studied the buckling behavior of generally laminated composite plate by using the generalized differential quadrature method and Rayleigh–Ritz method. Comparisons of the GDQ results with Rayleigh–Ritz results were carefully studied. The influence of the fiber orientation on buckling load was also studied. Jensen and Lagace [3] performed an experimental and analytical investigation on the bucking and post-buckling of generally an-isotropic laminated plate. The Rayleigh–Ritz and the finite element methods were used to predict the buckling loads; different an-isotropic couplings inherent in unbalanced and un-symmetric laminates were

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isolated and their effects were studied. Kabir and Chaudhuri [4,5] developed a boundary continuous generalized Navier's approach, and presented an analytical solution for free vibration of arbitrarily laminated plate with clamped and simply supported boundary conditions, in which the effect of shear deformation was considered.

In this paper, free vibrations of an arbitrarily laminated rectangular plate is studied by employing a novel numerical solution technique, the moving least squares differential quadrature (MLSDQ) method. As an efficient and accurate global solution technique, the differential quadrature (DQ) method were first introduced by Bellman and his associates [6,7] for solving linear and nonlinear differential equations with a little computational cost. Since then, there have been numerous developments and applications of the method in structural mechanics [8–10]. However, further application of the method has been greatly restricted by the disadvantage that it cannot be directly used to solve problems with discontinuities or with complex domains, since the grid points used in DQ method must be distributed in a regular manner in order to express the weighting coefficients explicitly. Although the domain decomposition can be used for discontinuous domains and the domain transformation is possible for irregular shapes, it may cause a significant loss of efficiency and simplicity, especially for problems involving irregular geometries and higher order partial derivatives. To overcome these drawbacks, Liew et al. [14,15] developed a new kind of numerical method, the MLSDQ method to solve the static and buckling problems of shear deformable plate. The MLSDQ method is a combination of the general differential quadrature method and the moving least-squares meshless method. In MLSDQ method, the weighting coefficients of quadrature approximation are calculated directly from the partial derivatives of shape functions used in the moving least squares elementfree method. Since the mesh points used in elementfree method can be arbitrarily located, the MLSDQ method can be easily used to solve problems having complex domains. In the present paper, this method is employed to study the free vibration problems of generally laminated composite plates having different boundary conditions. The suitability, efficiency, simplicity and convergence properties of this method are all demonstrated. The numerical accuracy is verified by the comparison of the present results with corresponding exact solutions or other numerical solutions in the open literature.

2. The MLSDQ method

Consider a domain in the space Ω discretized by a set of discrete points $\{x_i\}_{i=1,2,\ldots,N}$. In the generalized differential quadrature (DQ) or the differential cubature (DC) method, the value of a partial derivative of a certain function $u(x)$ at a discrete point x_i can be approximate as a weighted linear sum of discrete function values chosen within the overall domain of a problem.

$$\hbar\{u(x)\}_i = \sum_{j=1}^{N} c_j(x_i)u(x_j) = \sum_{j=1}^{N} c_{ij}u_j, \tag{1}$$

where $\hbar$ denotes a linear differential operator which can be any orders of partial derivatives or their combinations, c_{ij} are the weighting coefficients, and u_j are the nodal function values. According to Civan [16] and Liew [9], the weighting coefficients c_{ij} can be determined by solving a set of linear algebraic equations which can be obtained by selecting N monomials from a set of polynomial basis and substituting them into Eq. (1). For regular node patterns, explicit expressions of weighting coefficients can be obtained for the first, second and higher derivatives using the Lagrangian interpolation polynomials.

In this paper, the weighting coefficients are directly computed from the partial derivatives of shape functions used in the moving least squares method. Following Belytschko et al. [17], we have

$$u^h(x) = \sum_{i=1}^{m} p_i(x)a_i(x) = \mathbf{p}^{\mathrm{T}}(x)\mathbf{a}(x), \tag{2}$$

where $p_i(x)$ is a finite set of basis functions and $a_i(x)$ are the unknown coefficients, m denotes the total number of basis functions. In this work on 2D problems, the intrinsic polynomial basis with $m = 6$ is quadratic, i.e.

$$\mathbf{p}^{\mathrm{T}}(x) = [1, x, y, x^2, xy, y^2]. \tag{3}$$

The coefficients $a_i(x)$ are functions of the spatial coordinates, and they can be obtained by minimizing a weighted, discrete L_2 normal defined as

$$J(\mathbf{a}) = \sum_{i=1}^{n} \bar{\omega}_i(x)[\mathbf{p}^{\mathrm{T}}(x_i)\mathbf{a}(x) - u_i]^2, \tag{4}$$

where n is the number of nodes in the neighborhood of x and u_i is the nodal parameter of $u(x)$ at point $x_i \cdot \bar{\omega}_i(x) = \bar{\omega}(x - x_i)$ is a positive weight function which decreases as $\|x - x_i\|$ increases. It always assumes unity at the sampling point x and vanishes outside the domain of influence of x. The size of the domain of influence, or support size, determines the number of discrete points n in the domain of influence.

The extremum of $J(\mathbf{a})$ with respect to $\mathbf{a}(x)$ results in the following linear equations

$$A(x)\mathbf{a}(x) = B(x)\mathrm{u} \tag{5}$$

from which

$$\mathbf{a}(x) = A^{-1}(x)B(x)\mathrm{u}, \tag{6}$$