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Analysis of laminated shear-flexible angle-ply plates

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Abstract

A recently developed C^0-type triangular composite plate element, based on the assumption of transverse inextensibility and layerwise constant shear-angle theory (LCST), is utilized to analyze antisymmetric and symmetric angle-ply plates subjected to distributed transverse loading. Effect of numerical integration on the rate of convergence of displacements and moments is investigated in detail. Comparison of the numerical results computed using the present triangular element with their analytical counterparts based on the classical lamination theory (CLT) in the thin plate regime also forms a major part of the present investigation. Limited comparisons with the first-order shear deformation theory (FSDT) results, computed using assumed stress hybrid finite elements, are also presented. Numerical results presented also include the effect of fiber orientation angle on the displacements and moments for thin laminates. Finally, the effect of thickness on the computed transverse displacement (deflection), interfacial inplane displacement and inplane stress is also thoroughly investigated.

Keywords: Anisotropic; Angle-ply; Composite; Laminate; Layerwise constant shear-angle theory (LCST); First-order shear deformation theory (FSDT); Classical lamination theory (CLT); Interlaminar shear deformation; Thick plate; Triangular element; Scaling effect

1. Introduction

Recent years have witnessed an increasing use of advanced composite materials (e.g., graphite/epoxy, boron/epoxy, Kevlar/epoxy, graphite/PEEK, etc.), which are replacing metallic alloys in the fabrication of load-bearing plate-type structures because of many beneficial properties, such as higher strength-to-weight ratios, longer fatigue (including sonic fatigue) life, better stealth characteristics, enhanced corrosion resistance, and, most significantly, the possibility of optimal design through the variation of stacking pattern, fiber orientation, and so forth, known as composite tailoring. The advantages that accrue from these properties are, however, not attainable without paying for the complexities that are introduced by various coupling effects. Furthermore, since the matrix material is of relatively low shearing stiffness as compared to the fibers, a reliable prediction of the response of these laminated shells must account for interlaminar (transverse) shear deformation or cross-sectional warping of individual layers, in contrast to the Kirchhoff or Mindlin hypothesis. The former, known as the classical lamination theory (CLT), neglects the interlaminar shear deformation altogether, while the latter, called the first-order shear deformation theory (FSDT) assumes constant transverse shear deformation through the entire thickness of the laminate. More recently, various refined or higher-order shear deformation theory (HSDT) based solutions have become available in the literature. Basset [1] appears to have been the first to suggest that the in-plane displacements can be expanded in power series of the thickness coordinate. Following Basset's lead, second- and higher-order shear deformation theories (HSDT), assuming transverse inextensibility and continuous inplane displacements through the thickness of thick laminates, have been developed as special cases of the above, e.g., Refs. [2,3]. A detailed review of the literature and a double Fourier series solution to a general type boundary-value problem pertaining to HSDT-based laminates are available in Chaudhuri and Kabir [4].

Noor and Burton [5] have presented extensive surveys on shear deformation theories and computational models relating to laminated plates. Exact three-dimensional elasticity solutions for rectangular cross-ply plates for a specific type of simply supported boundary condition are due to Pagano [6], and Srinivas and Rao [7]. Approximate thick laminate theories can be classified

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Nomenclature

a, b	length and width of a laminated plate
$[B_j]$	strain–nodal displacement relation matrix for the jth composite plate element
$[C^{(i)}]$, $[G^{(i)}]$	elastic stiffness matrices of the ith anisotropic lamina in inplane stretching/shear and transverse shear, respectively
$\{d_j\}$	nodal displacement vector of the jth composite plate element
$\bar{d}_i$	distance from the bottom (reference) surface
E_1, E_2	Young's moduli of an orthotropic lamina in the direction of fibers and normal to the fibers, respectively
F_j	consistent load vector of the jth composite plate element
G_{12}	inplane shear modulus of an orthotropic lamina
G_{13}, G_{23}	transverse shear moduli of an orthotropic lamina
$[K_j]$	stiffness matrix for the jth composite plate element
N	total number of layers or laminae
N_d	number of subdivisions in a finite element model
q_0	applied uniformly distributed load (normal pressure) on the top surface of a laminated plate
$q_j(x_1, x_2)$	applied surface load on the top surface of the jth composite plate element
t_i, t	thickness of the ith lamina and laminated plate, respectively
U	strain energy of a laminated plate
$\overline{U}$	strain energy per unit area
$U_{\mathrm{B}}^{(i)}$	strain energy in bending and twisting of the ith layer
$U_{\mathrm{S}}^{(i)}$	strain energy in transverse (interlaminar) shear of the ith layer
u_i	components of the displacement vector, $i = 1, 2, 3$
W	potential due to external conservative forces
w	deflection of a plate
x_i	cartesian coordinates, $i = 1, 2, 3$
$\gamma_{12}^{(i)}$	inplane (engineering) shearing strain at a point inside the ith layer
$\gamma_{13}^{(i)}$, $\gamma_{23}^{(i)}$	transverse (engineering) shearing strains at a point inside the ith layer
Δ_j	reference surface area of the jth element
$\varepsilon_{k\ell}^{(i)}$	inplane components of the strain tensor at a point inside the ith layer; $k, \ell = 1, 2$
ν_{12}, ν_{13}, ν_{23}	major Poisson's ratios of an orthotropic lamina
Π	total potential energy functional
$\bar{\tau}_{13}^{(N+1)}$, $\bar{\tau}_{23}^{(N+1)}$, $\bar{\sigma}_{33}^{(N+1)}$, $\bar{\tau}_{13}^{(1)}$, $\bar{\tau}_{23}^{(1)}$, $\bar{\sigma}_{33}^{(1)}$	applied distributed forces over the top and bottom surfaces of a N-layer plate
$\bar{\sigma}_{nn}^{(i)}(x_3)$, $\bar{\tau}_{n\Gamma}^{(i)}(x_3)$, $\bar{\tau}_{n3}^{(i)}(x_3)$	applied forces at a boundary distributed through the thickness of the ith layer
$\{\phi\}$	shape functions

into two categories: (a) discrete layer approach, and (b) continuous inplane displacement through thickness, e.g. the CLT, FSDT and HSDT, mentioned above. The former approach, e.g., the layerwise constant shear-angle theory (LCST) or the zig-zag theory, first introduced by Mau et al. [8] appears to be quite suitable for numerical methods, such as the degenerate type finite element methods (FEM), although a number of analyses on shear-flexible laminated plates have been performed using FSDT-based finite elements, e.g., Spilker et al. [9]. Seide [10] has utilized the LCST in the derivation of an exact solution to an infinitely long laminated strip problem.

The LCST-based solution due to Mau et al. [8] for laminated thick plates has used a quadrilateral element shape, and assumed stress hybrid finite element method (FEM). Although this method has yielded results for certain simpler laminated plate problems, it suffers from certain difficulties, such as the presence of spurious kinematic modes [11]. More recently, Chaudhuri and Seide [12], and Seide and Chaudhuri [13] have developed a quadratic triangular element, based on the LCST, and an assumed displacement potential energy approach, for analyses of laminated plates and shells, respectively. Example problems analyzed by Chaudhuri and Seide [12] include a symmetric cross-ply [90°/0°/90°] infinitely long strip and a three-layer symmetric cross-ply square plate. Additionally, the special case of a homogeneous isotropic plate has been presented by Chaudhuri [14]. LCST-based results on angle-ply plates, computed using this triangular element, are not available in the literature, which is the primary objective of the present investigation. The specific goals of the present analysis, which assumes quadratic shape functions, are to (i) obtain satisfactory convergence of displacements and moments, (ii) study the effect of triangulation pattern on convergence, (iii) investigate the effect of numerical integration on convergence, and (iv) study the effects of fiber orientation angle and length-to-thickness ratio on computed displacements and stresses (or moments).