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Composite Structures 69 (2005) 421–428
COMPOSITE STRUCTURES
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Buckling of thin-walled composite structures with intermediate stiffeners

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Available online 27 August 2004

Abstract

The interactive buckling of prismatic, thin-walled composite columns with open sections, reinforced with intermediate stiffeners and with edge reinforcements, has been considered. The columns are assumed to be simply supported. The nonlinear problem has been solved with the Koiter's asymptotic theory within the first order approximation. The asymptotic theory of the first order non-linear approximation allows for simultaneous evaluation of the effect of imperfections and interactions of various modes of buckling on the behaviour of thin-walled structures. This evaluation can be only the lower bound estimation of the load carrying capacity. Detailed calculations have been made for several cases of columns.
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Keywords: Composite; Buckling; Stiffeners; Imperfection; Interaction; Multi-layer plates

1. Introduction

Since their appearance in the 1960s, modern composite materials have been very widely used in modern structures, whereas the progress in their applications depends extensively on the knowledge and capabilities of designers and users. The strength of composites along the direction of the reinforcing fibre alignment is several times higher than along other directions. Therefore, they require much more attention during the designing process than isotropic structures. A better understanding of phenomena that occur during the stability loss and afterwards is necessary to evaluate correctly the performance of composite thin-walled structures.

The authors of the present paper have been investigating the nonlinear stability of isotropic thin-walled structures with intermediate stiffeners for almost ten years. Their investigations have been summed up in [10]. There, the evaluation of the load carrying capacity of composite thin-walled structures with open sections, reinforced with intermediate stiffeners, has been presented. The stiffeners carry a portion of loads and subdivide the plate element into smaller parts of higher stiffness, thus increasing significantly their load carrying capacity. The size, shape and position of intermediate stiffeners and edge reinforcements exert a strong influence on the critical and post-buckling behaviour of the structure. The asymptotic theory of the first order non-linear approximation [1–3,5–8] allows for simultaneous evaluation of the effect of imperfections and interactions of different buckling modes on the behaviour of thin-walled structures. It can be only the lower bound estimation of the load carrying capacity or a trial to estimate the load carrying capacity within the second order on the basis of the approximation assumed for the linear analysis of stability [4].

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doi:10.1016/j.compstruct.2004.07.022

2. Formulation of the problem

Thin-walled composite columns with open sections, subjected to compression, have been analysed. The cross-sections of the elements under analysis are built of rectangular plates of the length L, which are connected along longitudinal edges and simply supported at both ends. To each ith plate, a local Cartesian system of coordinates has been assigned. The material all plates are made of is subjected to the Hooke's law. In the theoretical analysis, the classical theory of composite plates [5,7] and the plate model of thin-walled structures have been assumed [8].

The following geometrical relationships have been assumed for each ith plate [5,7]:

$$\begin{aligned} \varepsilon_1 &= u_{1,1} + 0.5u_{m,1}u_{m,1} \qquad & \varepsilon_4 &= -hu_{3,11} \\ \varepsilon_2 &= u_{2,2} + 0.5u_{m,2}u_{m,2} \qquad & \varepsilon_5 &= -hu_{3,22} \\ \varepsilon_3 &= u_{1,2} + u_{2,1} + u_{m,1}u_{m,2} \qquad & \varepsilon_6 &= -2hu_{3,12} \end{aligned} \tag{1}$$

where h is the plate thickness, $u_1 \equiv u$, $u_2 \equiv v$, $u_3 \equiv w$—components of the displacement vector with respect to the axis $x_1 \equiv x$, $x_2 \equiv y$, $x_3 \equiv z$ and $\varepsilon_1 \equiv \varepsilon_x$, $\varepsilon_2 \equiv \varepsilon_y$, $\varepsilon_3 \equiv 2\varepsilon_{xy} = \gamma_{xy}$, $\varepsilon_4 \equiv h\kappa_x$, $\varepsilon_5 \equiv h\kappa_y$, $\varepsilon_6 \equiv h\kappa_{xy}$. The summation with respect to m=1, 2, 3.

Employing the classical theory of multi-layer plates, the constitutive equations have the following form [5,7]:

$$\{N\} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} = [K]\{\varepsilon\}, \tag{2}$$

where N_1, N_2, N_3 are the dimensionless cross-sectional forces, whereas N_4, N_5, N_6 are the dimensionless cross-sectional moments:

$$\begin{aligned} N_1 &= \frac{N_x}{E_0 h}, \qquad N_2 = \frac{N_y}{E_0 h}, \qquad N_3 = \frac{N_{xy}}{E_0 h}, \\ N_4 &= \frac{M_x}{E_0 h^2}, \qquad N_5 = \frac{M_y}{E_0 h^2}, \qquad N_6 = \frac{M_{xy}}{E_0 h^2} \end{aligned} \tag{3}$$

where E_0 is the Young modulus of reference.

From the principle of virtual work for a single ith plate, taking into consideration relationships (1), the following differential equations of equilibrium results:

$$\begin{aligned} &[N_1(1+u_{1,1}) + N_3 u_{1,2}]_{,1} + [N_2 u_{1,2} + N_3(1+u_{1,1})]_{,2} = 0 \\ &[N_1 u_{2,1} + N_3(1+u_{2,2})]_{,1} + [N_2(1+u_{2,2}) + N_3 u_{2,1}]_{,2} = 0 \\ &(hN_{4,1} + N_1 u_{3,1} + N_3 u_{3,2})_{,1} \\ &\quad + (hN_{5,2} + 2hN_{6,1} + N_2 u_{3,2} + N_3 u_{3,1})_{,2} = 0 \end{aligned} \tag{4}$$

The solution to system (4) for the ith plate that forms a thin-walled column must fulfil the kinematics and static conditions of co-operation along the longitudinal edges and additionally, the boundary conditions corresponding to a simple support on both column ends (for $x_1 = 0; L$ [9]). The nonlinear problem of stability within the first order nonlinear approximation has been solved with the asymptotic Byskov–Hutchinson's method [1]. The fields of displacements $\overline{U}$ and the fields of forces $\overline{N}$ have been expanded into the power series with respect to the dimensionless amplitudes of buckling mode ξ_j:

$$\overline{U} = \lambda\overline{U}^{(0)} + \xi_j\overline{U}^{(j)} + \cdots \qquad \overline{N} = \lambda\overline{N}^{(0)} + \xi_j\overline{N}^{(j)} + \cdots \tag{5}$$

where $\overline{U}^{(0)}$, $\overline{N}^{(0)}$—pre-buckling fields, $\overline{U}^{(j)}$, $\overline{N}^{(j)}$—critical states. The range of indices j is equal to $[1, J]$, where J is the number of interacting buckling modes.

At the point, where the load parameter λ reaches the maximum value λ_s for structures with imperfections, the Jacobean of the nonlinear system of equations [1,5–8]:

$$a_r\left(1 - \frac{\lambda}{\lambda_r}\right)\xi_r + a_{jkr}\xi_j\xi_k + b_{jklr}\xi_j\xi_k\xi_l = a_r\frac{\lambda}{\lambda_r}\xi_r^* \quad \text{for } r = 1, 2, \ldots, J \tag{6}$$

is equal to zero. The expression for the total potential energy corresponding to (6) can be written as follows:

$$\begin{aligned} \Pi = &-a_0\lambda^2/2 + a_r(1 - \lambda/\lambda_r)\xi_r^2/2 + a_{jkr}\xi_j\xi_k\xi_r/3 \\ &+ b_{jklr}\xi_j\xi_k\xi_l\xi_r/4 - a_r\xi_r\xi_r^*\lambda/\lambda_r \end{aligned} \tag{7}$$

where $\Pi_0 = a_0\lambda^2/2$—energy of the pre-buckling state.

The coefficient of the reduced rigidity, corresponding to the kth uncoupled local buckling mode for a structure without initial deflections and with the symmetrical characteristics of deflections for the uncoupled local buckling mode ($a_{kkk} = 0$), is described by the following relationship [4,6,8]:

$$\bar{\eta}_k = \lim_{x\to\infty}\eta_k = \left[1 + \frac{a_k^2}{2a_0 b_{kkkk}}\right]^{-1} \approx \left[1 - \frac{a_k^2}{2a_0\bar{b}_{kkkk}}\right] \tag{8}$$

The approximate values of b_{kkkk} for the kth uncoupled local mode can be determined from formula (8):

$$b_{kkkk} = \bar{b}_{kkkk} - \frac{a_k^2}{2a_0} \tag{9}$$

where the value of $\bar{b}_{kkkk}$ is determined on the basis of the knowledge of the function of deflection of the kth uncoupled local buckling mode for the first order approximation [4].

For a single plate, the coefficient of the reduced rigidity is limited to the known formulae of Koiter and Pignataro [3]. A detailed description of the solution method to the problem is included in [4]. Further on, it has been assumed that $J = 2$ and that 1 denotes the global mode of buckling, 2 the local mode of buckling.

3. Analysis of the calculation results

Below, the results of calculations for the columns under compression, shown in Fig. 1, of the following geometrical dimensions: $b_1 = 50$ mm, $b_2 = 25$ mm,

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