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A general solution for a bolt loaded-hole in piezoelectric composite laminates

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Abstract

In this paper the two-dimensional problem of a bolt loaded-hole in an infinite piezoelectric laminate is considered in terms of the complex variable method. Firstly, an explicit form Green function for a generalized point load acted at an arbitrary point outside the hole is derived. Secondly, the Green function for a generalized point load acted at the rim of the hole, as a special case, is obtained, and then a general solution for the case of arbitrarily distributed mechanical and electric loading on the hole surface is presented based on the superposition principle. In general, the solution is in the form of series, but its novel feature is that the coefficients involved in the solution can be easily written out once the distributed loading is specified. Finally, several examples useful for engineering are given.

Keywords: Piezoelectric composites; Circular hole; Stress concentration; General solution

1. Introduction

Piezoelectric composite laminates are being increasingly used in aeronautical and aerospace structures. Holes are usually unavoidable in these structural designs. The existence of holes causes geometric discontinuities and induces high stress concentrations, which may finally leads to fracture/failure of structures. Thus, the analysis of local field concentrations around holes in piezoelectric materials is very important for the reliability design of smart structures. However, piezoelectric works were mainly restricted to the cases of free hole under remote-field loadings [1,2]. In fact, it is more important to study the cases where a hole is loaded on its surface. In the assembly of composite structures, for example, bolted joints are common practice to fasten parts in place and transfer load from one component to another. Since the bolt loaded-holes lead to stress concentrations that can result in premature failure of the structure, it is necessary to develop appropriate methods for stress analysis of bolt loaded-holes. Extensive theoretical and experimental studies on the strength predications of composite laminates with bolt loaded-holes have indeed been carried out in recent decade [3–7], but they are restricted to the cases of purely elastic materials.

In the present work, a general solution for stress analysis of piezoelectric composite laminates with a bolt loaded-hole based on complex variable method is developed. Firstly, an explicit form Green function for a generalized line load acted at an arbitrary point on the surface of the hole is derived. Secondly, a general solution for the case of arbitrarily distributed mechanical and electric loading on the surface of the hole is given. In general, the solution can be expressed in series form, but its novel feature is that the coefficients involved in

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the solution can be easily written out provided that the loading distribution on the hole surface is specified. Finally, several examples useful for engineering are presented.

2. Basic equation

Consider a symmetric laminate of piezoelectric composite that comprises piezo-ceramic fiber composite plies and polymeric matrix material piles. It is assumed that the laminate is in the state of a two-dimensional deformation under the mechanical and electric loading in the x_1–x_2 plane, that is:

$$\varepsilon_{33} = \varepsilon_{23} = \varepsilon_{13} = E_3 = 0, \tag{1}$$

where ε_{ij} stand for the strain and E_3 is the component of electric field. Furthermore, assume that the constitutive equations can be characterized by

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} + \begin{bmatrix} 0 & b_{21} \\ 0 & b_{22} \\ b_{13} & 0 \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix}, \tag{2}$$

$$\begin{Bmatrix} E_1 \\ E_2 \end{Bmatrix} = -\begin{bmatrix} 0 & 0 & b_{13} \\ b_{21} & b_{22} & 0 \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} + \begin{bmatrix} \delta_{11} & 0 \\ 0 & \delta_{22} \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix}, \tag{3}$$

where σ_{ij} are the stresses, D_i are the electric displacements, and a_{ij}, b_{ij} and δ_{ii} represent the reduced elastic constants, piezoelectric constants and dielectric constants, respectively.

Equations of generalized strains are

$$\begin{aligned} &\varepsilon_{11} = \frac{\partial u_1}{\partial x_1}, \quad \varepsilon_{11} = \frac{\partial u_2}{\partial x_2}, \quad 2\varepsilon_{12} = \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}, \\ &E_1 = -\frac{\partial \phi}{\partial x_1}, \quad E_2 = -\frac{\partial \phi}{\partial x_2}, \end{aligned} \tag{4}$$

where u_i are the components of displacement and ϕ is the electric potential.

Equations of generalized equilibrium are

$$\begin{aligned} &\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{21}}{\partial x_2} = 0, \quad \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{12}}{\partial x_1} = 0, \\ &\frac{\partial D_1}{\partial x_1} + \frac{\partial D_2}{\partial x_2} = 0. \end{aligned} \tag{5}$$

Following Sosa's work [1], the general solution for Eqs. (2)–(5) can be expressed as

$$\begin{aligned} &\langle \sigma_{22}, -\sigma_{12}, \sigma_{11} \rangle = 2\mathbf{Re} \sum_{k=1}^{3} \langle 1, \mu_k, \mu_k^2 \rangle \phi_k(z_k), \\ &z_k = x_1 + \mu_k x_2, \end{aligned} \tag{6}$$

$$\langle u_1, u_2 \rangle = 2\mathbf{Re} \sum_{k=1}^{3} \langle p_k, q_k \rangle \varphi_k(z_k), \tag{7}$$

$$\langle E_1, E_2 \rangle = 2\mathbf{Re} \sum_{k=1}^{3} \kappa_k \langle 1, \mu_k \rangle \phi_k(z_k), \tag{8}$$

$$\langle D_1, -D_2 \rangle = 2\mathbf{Re} \sum_{k=1}^{3} \lambda_k \langle \mu_k, 1 \rangle \phi_k(z_k), \tag{9}$$

$$-\phi = 2\mathbf{Re} \sum_{k=1}^{3} \kappa_k \varphi_k(z_k), \tag{10}$$

where **Re** denotes taking the real part, μ_k are distinct complex parameters [1], $\varphi_k(z_k)$ are unknown complex potentials, $\phi_k(z_k) = \varphi_k(z_k)/dz_k$, and

$$p_k = a_{11}\mu_k^2 + a_{12} - b_{12}\lambda_k, \quad q_k = \left(a_{12}\mu_k^2 + a_{22} - b_{22}\lambda_k\right)/\mu_k,$$

$$\lambda_k = -\frac{(b_{21} + b_{13})\mu_k^2 + b_{22}}{\delta_{11}\mu_k^2 + \delta_{22}}, \quad \kappa_k = (b_{13} + \delta_{11}\lambda_k)\mu_k.$$

When the generalized traction is specified on the boundary, $\varphi_k(z_k)$ can be determined from the following boundary equations:

$$2\mathbf{Re} \sum_{k=1}^{3} \varphi_k(z_k) = -\int_0^s t_2 \, \mathrm{d}s, \tag{11}$$

$$2\mathbf{Re} \sum_{k=1}^{3} \mu_k \varphi_k(z_k) = \int_0^s t_1 \, \mathrm{d}s, \tag{12}$$

$$2\mathbf{Re} \sum_{k=1}^{3} \lambda_k \varphi_k(z_k) = -\int_0^s D_n \, \mathrm{d}s, \tag{13}$$

where t_1 and t_2 represent the force components along the x_1 and x_2 axes, respectively; s is the arc-length on the boundary, and D_n is the normal component of electric displacement.

Introduce the matrices defined as

$$\begin{aligned} &\boldsymbol{\varphi}(z) = \begin{Bmatrix} \varphi_1(z_1) \\ \varphi_2(z_2) \\ \varphi_3(z_3) \end{Bmatrix}, \quad \mathbf{u} = \begin{Bmatrix} u_1 \\ u_2 \\ -\phi \end{Bmatrix}, \quad \mathbf{t}_s = \begin{Bmatrix} t_2 \\ -t_1 \\ D_n \end{Bmatrix}, \\ &\mathbf{A} = \begin{bmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \\ \kappa_1 & \kappa_2 & \kappa_3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 1 \\ \mu_1 & \mu_2 & \mu_3 \\ \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix}. \end{aligned} \tag{14}$$

Then, we have form (7) and (10), and (11)–(13) that

$$\mathbf{A}\boldsymbol{\varphi}(z) + \overline{\mathbf{A}}\,\overline{\boldsymbol{\varphi}(z)} = \mathbf{u}, \tag{15}$$

$$\mathbf{B}\boldsymbol{\varphi}(z) + \overline{\mathbf{B}}\,\overline{\boldsymbol{\varphi}(z)} = -\int_0^s t_s \, \mathrm{d}s. \tag{16}$$