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Static analysis of functionally graded plates using third-order shear deformation theory and a meshless method

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Abstract

The collocation multiquadric radial basis functions are used to analyze static deformations of a simply supported functionally graded plate modeled by a third-order shear deformation theory. The plate material is made of two isotropic constituents with their volume fractions varying only in the thickness direction. The macroscopic response of the plate is taken to be isotropic and the effective properties of the composite are derived either by the rule of mixtures or by the Mori–Tanaka scheme. Effects of aspect ratio of the plate and the volume fractions of the constituents on the centroidal deflection are scrutinized. When Poisson's ratios of the two constituents are nearly equal, then the two homogenization techniques give results that are close to each other. However, for widely varying Poisson's ratios of the two constituents, the two homogenization schemes give quite different results. The computed results are found to agree well with the solution of the problem by an alternative meshless method.

Keywords: Static analysis; Functionally graded materials; Thick plate; Meshless methods; Multiquadric radial basis functions

1. Introduction

An advantage of a plate made of a functionally graded material (FGM) over a laminated plate is that material properties vary continuously in a FGM but are discontinuous across adjoining layers in a laminated plate. It eliminates at least the delamination mode of failure. Furthermore, in an FGM, material properties can be tailored to optimize the desired characteristics, e.g., minimize the maximum deflection for a given type of loads and boundary conditions, or maximize the first frequency of free vibration of the structure. Even though material properties may vary continuously in all three directions, here we limit ourselves to analyzing static deformations of a FG plate with material properties varying only in the thickness direction.

Several investigators, e.g., see [1–4], have analyzed deformations of a FG plate either by using a plate theory or three-dimensional equations of linear elasticity for an inhomogeneous body. Exact solutions for static and dynamic deformations of a FG plate are given in [5–8]. Here we use a meshless method and a third-order shear deformation plate theory. We note that Qian et al. [9–11] used the meshless local Petrov–Galerkin method (MLPG) and either two-dimensional equations of thermoelasticity or a higher-order shear and normal deformable plate theory of Batra and Vidoli [12] to analyze static and dynamic deformations of a FG plate. The MLPG method does not need even a background mesh but requires integration over a local subdomain and the determination of basis functions by say the moving least

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squares method [13]. Thus, it is computationally expensive. Here we use the collocation method and the multiquadric radial basis functions which neither require a mesh nor the numerical evaluation of integrals over any subdomain. The goal here is to show that this meshless method gives results close to the analytical solution of the problem for a FG plate. No attempt has been made to review all of the literature on meshless methods, plate theories, homogenization techniques to deduce effective properties of a composite from those of its constituents, methods of manufacturing a FG plate, and papers dealing with the analysis of FG plates.

Meshless methods for finding an approximate solution of a boundary-value problem include the element-free Galerkin [14], hp-clouds [15], the reproducing kernel particle [16], the smoothed particle hydrodynamics [17], the diffuse element [18], the partition of unity finite element [19], the natural element [20], meshless Galerkin using radial basis functions [21], the meshless local Petrov–Galerkin [22], the collocation technique employing radial basis functions [23], and the modified smoothed particle hydrodynamics [24]. Of these, the last three and the smoothed particle hydrodynamics method do not require any mesh whereas others generally need a background mesh for the evaluation of integrals appearing in the weak formulation of the problem. Ferreira [25,26] has used the collocation method with the radial basis functions to analyze several plate and beam problems. The applicability of the method is extended here to analyze static deformations of a thick FG plate with a third-order shear deformation plate theory (TSDT).

The paper is organized as follows. Section 2 briefly reviews the finite point multiquadric method of solving an elliptic linear boundary-value problem. Equations for a TSDT are derived in Section 3, and two homogenization techniques for determining effective material properties of a composite are summarized in Section 4. Section 5 discusses results and Section 6 gives conclusions.

2. The finite point multiquadric method

Consider the following linear elliptic boundary-value problem defined on a smooth domain Ω:

$$\begin{aligned} Lu(\mathbf{x}) &= s(\mathbf{x}), \quad \mathbf{x} \in \Omega, \\ Bu(\mathbf{x}) &= f(\mathbf{x}), \quad \mathbf{x} \in \partial\Omega, \end{aligned} \tag{2.1}$$

where $\partial\Omega$ is the boundary of Ω, L and B are linear differential operators, and s and f are smooth functions defined on Ω and $\partial\Omega$ respectively. We select N_B points ($\mathbf{x}^{(j)}$, $j = 1, \ldots, N_B$) on $\partial\Omega$ and $(N - N_B)$ points ($\mathbf{x}^{(j)}, j = N_B + 1, N_B + 2, \ldots, N$) in the interior of Ω. Let

$$u^h(\mathbf{x}) = \sum_{j=1}^{N} a_j g(\|\mathbf{x} - \mathbf{x}^{(j)}\|, c) \tag{2.2}$$

be an approximate solution of the boundary-value problem where $a_1, a_2, \ldots, a_N$ are constants to be determined, $\|\mathbf{x} - \mathbf{x}^{(j)}\|$ is the Euclidean distance between points $\mathbf{x}$ and $\mathbf{x}^{(j)}$, c is a constant, and g is a function of $\|\mathbf{x} - \mathbf{x}^{(j)}\|$ and c. Different forms of functions g and names associated with them are

$$\begin{aligned} &\text{Multiquadrics:} \\ &g_j(\mathbf{x}) = (\|\mathbf{x} - \mathbf{x}^{(j)}\|^2 + c^2)^{1/2}, \\ &\text{Inverse Multiquadrics:} \\ &g_j(\mathbf{x}) = (\|\mathbf{x} - \mathbf{x}^{(j)}\|^2 + c^2)^{-1/2}, \\ &\text{Gaussian:} \\ &g_j(\mathbf{x}) = \mathrm{e}^{-c^2\|\mathbf{x} - \mathbf{x}^{(j)}\|^2}, \\ &\text{Thin plate splines:} \\ &g_j(\mathbf{x}) = \|\mathbf{x} - \mathbf{x}^{(j)}\|^2 \log \|\mathbf{x} - \mathbf{x}^{(j)}\|. \end{aligned} \tag{2.3}$$

Substitution from (2.2) into (2.1) and evaluating the resulting form of Eq. $(2.1)_2$ at the N_B points $\mathbf{x}^{(j)}$, $j = 1, 2, \ldots, N_B$, and of Eq. $(2.1)_1$ at $(N - N_B)$ points $\mathbf{x}^{(j)}$, $j = N_B + 1, N_B + 2, \ldots, N$ give the following N algebraic equations for the determination of $a_1, a_2, \ldots, a_N$.

$$\begin{aligned} &\sum_{j=1}^{N} a_j L g(\|\mathbf{x} - \mathbf{x}^{(j)}\|, c)\Big|_{\mathbf{x}=\mathbf{x}^{(i)}} = s(\mathbf{x}^{(i)}), \\ &\quad i = N_B + 1, N_B + 2, \ldots, N, \\ &\sum_{j=1}^{N} a_j B g(\|\mathbf{x} - \mathbf{x}^{(j)}\|, c)\Big|_{\mathbf{x}=\mathbf{x}^{(i)}} = f(\mathbf{x}^{(i)}), \\ &\quad i = 1, 2, \ldots, N_B. \end{aligned} \tag{2.4}$$

Depending upon the value of the parameter c and the form of function g, the set of Eqs. (2.4) that determines $a_1, a_2, \ldots, a_N$ may become ill-conditioned; e.g. see [27]. Also, the computational effort involved in solving (2.4) for $a_1, a_2, \ldots, a_N$ varies with the choice of the function g. Once Eqs. (2.4) have been solved for a's, then the approximate solution of the problem is given by (2.2).

3. Review of the third-order shear deformation plate theory

The displacement field in the TSDT is given by

$$\begin{aligned} (x,y,z) &= u_0(x,y) + z\phi_x - c_1 z^3\left(\phi_x + \frac{\partial w}{\partial x}\right), \\ v(x,y,z) &= v_0(x,y) + z\phi_y - c_1 z^3\left(\phi_y + \frac{\partial w}{\partial y}\right), \\ w(x,y,z) &= w_0(x,y), \end{aligned} \tag{3.1}$$

where $c_1 = 4/(3h^2)$, h is the plate thickness, z is the coordinate in the thickness direction, and the xy-plane of the rectangular Cartesian coordinate system is located in the midplane of the plate. Functions ϕ_x and ϕ_y describe