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Diffusion of moisture into building materials: A model for moisture transport

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ABSTRACT

Presence of moisture in the building materials leads to a larger energy loss, especially in winter; therefore, drying is important to improve the energy efficiency of buildings. The drying process is based on the evaporation of the liquid water on the material surface where the water vapor releases to surrounding, or it is based on the evaporation in the pores of the material and subsequently moving through the pores to the surface and surrounding. We derive a mathematical model that describes liquid water and vapor diffusion in a wet material as two separate processes. We also present an exact solution of this model and compare it with the classical moisture transfer solution representing transfer of both liquid water and vapor as a single moisture variable. Finally, we present the calculation of diffusion coefficient and compile the values for various building materials. The model allows considering the drying in various materials as two independent processes for transfer of liquid water and vapor. Energy losses can be calculated using the model depending on the moisture content in the materials.

1. Introduction

Compared with many liquids, the heat capacity of liquid water is relatively large [1]. While the larger heat capacity of water sometimes plays a positive role, for example in nuclear industry, moisture transfer and buildup in the pores of building materials negatively affect the building performance. The effect of moisture transfer and subsequent undesirable thermal energy loss can be minimized when the building material is dry and moisture penetration is restricted. To achieve appropriate energy saving, the building insulations have been developed and presently many different insulation materials are used. The heat and moisture properties of the buildings and materials are studied in laboratory and also in situ [2,3]. Several models and computer programs presenting moisture and heat transfer in building elements have been developed [4] that some of them are commercially available (for example, Match, WUFI, Delphin) [5–8].

Because of these matters, the knowledge about the mechanism of moisture transfer in the porous media is important and useful in buildings considering all physical parameters and local conditions at an early stage. Moisture transfer in the building materials has been modeled in various ways. There are many models that moisture is considered in the form of the water content [9–12] in which the presence of water vapor has been neglected. Conversely, there are diffusion and penetration models that consider the water vapor and water liquid is neglected [13]. An experimental resolution between liquid flow and water vapor flow by diffusion in some porous materials is difficult because these materials have a very complex porous system. Thus vapor and liquid flow can be treated as parallel processes. In [14] the corresponding moisture flow is expressed as the summation of two transport equations, one using water vapor pressure to drive water vapor flow by diffusion, and the other using either capillary suction or relative humidity to drive moisture flow.

In this paper, we formulate a mathematical model that considers two separate transfer equations that are tied by a moisture source/sink function S (kg/s m^3) which expresses a rate of phase exchange of liquid to saturated water vapor or vice versa. One equation uses saturated water vapor concentration w_v (kg/m^3) to

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drive water vapor flow by diffusion with diffusion coefficients D_v (m^2/s), and the other uses liquid water concentration w_l (kg/m^3) to drive moisture flow with diffusion coefficients D_l (m^2/s). We also consider an experiment at ambient temperature and pressure conditions. A sample of a porous material is supposed to be saturated with liquid water in the beginning. The ambient condition in the environment creates a driving force within the material for moisture movement which we model by means of the corresponding boundary conditions. The real experiments showed that the sample dries in a long period, approximately 20 or 30 days, depending on the form, dimensions or porosity of the material [15,16].

In this paper, we model these phenomena through the diffusion properties of the material with different values of the diffusion coefficients for liquid water D_l, and saturated water vapor D_v, assuming the ambient conditions are held in the surrounding environment. Additionally, we present an exact solution of the model and compare it with the solution of the classical moisture transfer with the single unknown of w (kg/m^3), representing moisture concentration in the pores of the material as both liquid and vapor at a specified diffusion coefficient of D (m^2/s) [17]. We applied the separation variables method [18] for obtaining exact solutions. Nevertheless, there is one problem on this way, while D and D_v are completely measurable diffusion coefficients; the coefficient D_l is not measurable. A reason for this observation is that the experiment with single liquid water transfer without vapor in the pores of material under ambient conditions is impossible. However, the diffusion coefficient D_l can be calculated using the calculation method, which we develop in this paper. Finally, we made a table for some building materials frequently used in practice, including a set of the corresponding calculated diffusion coefficients D_l. We also study the corresponding dependencies due to the increasing number of new applications [16,19–21].

2. Modeling

2.1. Model I

For modeling, we consider the moisture transfer in the sample of a porous wet material with dimensions of 3 cm × 9 cm × 12 cm, according to method reported literature [15]. The sample is sealed on with a self-adhesive aluminum tape from all sides except the right side with a size of 3 cm × 12 cm, which is left open. As a consequence, the moisture diffusion occurs only in one direction of x along the width of the sample $0 \le x \le l$, where $l = 9$ cm.

The moisture transfer through the material can be presented with the following two partial differential diffusion equations:

$$\frac{\partial w_l}{\partial t} = \frac{\partial}{\partial x}\left(D_l \frac{\partial w_l}{\partial x}\right) - S, \quad 0 < x < l,\ t > 0 \tag{1}$$

$$\frac{\partial w_v}{\partial t} = \frac{\partial}{\partial x}\left(D_v \frac{\partial w_v}{\partial x}\right) + S, \quad 0 < x < l,\ t > 0. \tag{2}$$

Additional condition [22] for porosity of the material leads to Eq. (3)

$$\Pi = \frac{w_l}{\rho_l} + \frac{w_v}{\rho_v}, \quad 0 \le x \le l,\ t \ge 0 \tag{3}$$

where ρ_i, D_i, $w_i = w_i\ (x;\ t)$ respectively present density, diffusion coefficient, and concentration for liquid ($i = l$) and vapor ($i = v$), and Π is the porosity of the material. The additional conditions of Eq. (3), used in Ref. [22], have a simple meaning; the pores volume is equal to sum of the liquid and vapor volumes (liquid-filled and vapor-filled pore fractions [14]). The function S in Eqs. (1) and (2) expresses a rate of variation of the moisture concentration that arises due to the evaporation ($S > 0$) of water in the pores of the material.

To solve the system of Eqs. (1)–(3), the following initial and boundary conditions were assumed:

$$w_l(x, 0) = \Pi\rho_l, \quad 0 \le x \le l \tag{4}$$

$$\frac{\partial w_l}{\partial x}(0, t) = 0, \quad t \ge 0 \tag{5}$$

$$w_l(1, t) = \Pi\rho_l + (v_0 - \Pi\rho_l)(1 - e^{-\alpha t}), \quad t \ge 0 \tag{6}$$

$$\rho_l > \rho_v > 0, \quad D_l > D_v > 0, \quad \Pi > 0, \quad \alpha \gg 1, \quad 0 < v_0 < \Pi\rho_l \tag{7}$$

for $0 \le t \le t_0$, where $t_0 = 20$ days. The initial boundary condition of Eq. (4) means that at the beginning, all pores of the sample are entirely filled with liquid (no vapor is present). There is no mass flux on the left boundary $x = 0$, presented as Eq. (5). The boundary condition in Eq. (6) is written on the right side of the sample when $x = l$, which describes the time dependences of the liquid concentration. The parameter α is introduced due to the consistency of the initial and boundary conditions and due to the smoothness with respect to time variable t of the liquid water at the boundary of $x = l$. In this system, v_0 represents a residual liquid concentration that is adopted from experiment [15].

2.2. Solution of model I

The system of Eqs. (1)–(3) with the initial conditions of (4) and the boundary conditions (5) and (6) has already been solved [18], resulting in the following exact solution:

$$w_l(x, t) = \Pi\rho_l + (v_0 - \Pi\rho_l)(1 - e^{-\alpha t}) + \sum_{k=0}^{\infty} T_k(t)X_k(x), \quad 0 \le x \le l,\ t \ge 0 \tag{8}$$

$$w_v(x, t) = \rho_v\left(\Pi - \frac{w_l(x, t)}{\rho_l}\right), \quad 0 \le x \le l,\ t \ge 0 \tag{9}$$

$$S = \frac{\rho_v(D_v - D_l)}{\rho_l - \rho_v}\sum_{k=0}^{\infty} \lambda_k T_k(t)X_k(x), \quad 0 < x < l,\ t > 0 \tag{10}$$

where

$$\lambda_k = -\left[\frac{2k+1}{2}\pi\right]^2, \quad X_k(x) = \cos\left[\frac{2k+1}{2}\pi x\right],$$

$$T_k(t) = \frac{g_k}{\alpha + \lambda_k \overline{D}}\left(e^{\lambda_k \overline{D} t} - e^{-\alpha t}\right) \tag{11}$$

$$g_k = \alpha(\Pi\rho_l - v_0)\frac{4(-1)^k}{(2k+1)\pi}, \quad k = 0, 1, 2, \ldots \quad \overline{D} = \frac{\rho_l D_l - \rho_v D_v}{\rho_l - \rho_v} \tag{12}$$

2.3. Model II and its solution

Let us consider the following classical model:

$$\frac{\partial w}{\partial t} = \frac{\partial}{\partial x}\left(D \frac{\partial w}{\partial x}\right), \quad 0 < x < l,\ t > 0 \tag{13}$$

$$w(x, 0) = \Pi\rho_l, \quad 0 \le x \le l \tag{14}$$

$$\frac{\partial w}{\partial x}(0, t) = 0, \quad t \ge 0 \tag{15}$$

$$w(l, t) = \Pi\rho_l + (v_0 - \Pi\rho_l)(1 - e^{-\alpha t}), \quad t \ge 0 \tag{16}$$