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Slenderness limit of the weak axis in the design of rectangular reinforced concrete non-sway columns

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ABSTRACT

Most design codes (Model Code 90, ACI-318-2008 and Eurocode–2-2004) suggest that the effect of a geometrical imperfection ought to be taken into account in the most unfavorable direction when designing slender reinforced concrete columns. If the bending plane of the beam–column is related to the strong axis, this imperfection acting in the opposite direction results in the column being subjected to axial load and biaxial bending, complicating the design procedure and increasing computational costs considerably. Such standards do not indicate when this geometrical imperfection generates a notable loss of resistance in the column. This paper proposes the geometrical slenderness of the weak axis (l_b/b) as the variable that determines whether or not to consider the influence of such imperfection. This paper denominates this boundary value the "slenderness limit of the weak axis" ($\lambda_{g,\text{weak}}$), and this is associated with a loss of resistance of 10% of the bending moment of the unbraced columns with respect to non-sway columns. An approximated equation of this limit is proposed using the results of a numerical simulation. The equation is valid for rectangular columns with doubly symmetric reinforcement and both normal and high strength concrete, and also for short-term and sustained loads.

1. Introduction

In general, reinforced concrete columns in buildings or industrial facilities are subjected to axial load and uniaxial bending moment. Cross sections are typically square or rectangular. The stiffness of the column in each bending direction varies due to the reinforcement distribution which is symmetric at opposite faces and parallel to the strong axis of bending, and is also different if the section is rectangular. This arrangement produces a strong axis perpendicular to the bending plane with higher stiffness than that of the weak axis parallel to the bending plane. Therefore, such columns are subjected to axial load and uniaxial bending with respect to the strong axis.

In this case, the behavior of the column can be affected by the flexibility of the weak axis, as a geometrical imperfection in the perpendicular direction to the principal bending plane means that the column is subjected to biaxial bending which reduces the load capacity of the column considerably. Furlong [1] stated that second order effects in columns subjected to biaxial bending are greatly influenced by the previous flexibility, producing a coupling of second-order effects from both bending directions. According to Bonet et al. [2], the rectangular columns subjected to uniaxial bending with respect to the strong axis, and with an axial load close to the critical axial compression of the column, experience an important loss of load capacity. Such authors affirm that the behavior of a column under axial load and uniaxial bending with respect to the strong axis depends on whether the bending of the column in the weak axis is neglected or not, Fig. 1(a). If the column is non-sway, the relative critical axial load corresponds to that of the strong axis ($\nu_{cr,\text{strong}}$). Moreover, if the column is unbraced and the eccentricity tends to zero, the behavior of the column is affected by the weak axis and the relative critical axial load corresponds to that of the weak axis ($\nu_{cr,\text{weak}}$). This behavior was experimentally observed by Pallarés et al. [3].

Menegotto [4] pointed out that when the geometrical slenderness of both axes of bending is very different ($\lambda_{g,\text{strong}} \neq \lambda_{g,\text{weak}}$, Fig. 1), the interaction of the weak axis of bending can cause an important reduction in the bending capacity of the column with respect to the strong axis, due to second-order effects. In this case, in order to analyze the behavior of the column, if the interaction diagram is linearized or simplified (as indicated in Eurocode 2 [5]) by calculating the first order relative bending moments of each bending direction in an uncoupled way in ($\mu_{xi,\text{strong}}$, $\mu_{yi,\text{weak}}$), an unsafe situation can result, Fig. 1(b). This author affirms that there is no criterion in the literature to evaluate or quantify the importance of this effect.

In fact, in the literature, most authors have studied the behavior of pinned–pinned columns with square sections subjected to biaxial bending. Only Mavichak and Furlong [6], Kim and Lee [7] and Pallarés et al. [3] have investigated the influence of the weak axis on the behavior of the column under biaxial bending.

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Notation

b	Width of the cross section.
h	Height (or the higher dimension of the section) of the cross section.
l_b	Effective buckling length.
ν	Relative axial load $= (N/f_c.b.h)$.
μ	Relative bending moment $= (M/f_c.b.h^2)$.
$\lambda_{g,\text{weak}}$	Geometrical slenderness of the weak axis $= (l_b/b)$.
$\lambda_{g,\text{strong}}$	Geometrical slenderness of the strong axis $= (l_b/h)$.
ω	Mechanical reinforcement ratio $= (A_s.f_s/A_c.f_c)$.

Mavichak and Furlong [8] tested 9 columns with geometrical slenderness with respect to the weak axis close to 14.9. Kim and Lee [7] performed experimental research on 10 columns with a slenderness close to 13, all with normal strength concrete. Pallarés et al. [3] tested 56 columns of high strength concrete with weak axis slenderness of 10, 20 and 30.

To design slender reinforced concrete columns subjected to axial load and uniaxial bending, the design codes (Model Code-90 [9], Eurocode 2 (2004) [5], ACI-318(08) [10]) propose different simplified methods to take second-order effects into account. Furthermore, these standards indicate that the effect of a geometrical imperfection must be taken into account in the most unfavorable direction when designing such columns.

The inclusion of this effect in the perpendicular bending plane complicates the design process when the column is subjected mainly to axial load and uniaxial bending with respect to the strong axis, due to the interaction that exists between the geometrical and the material non-linearity. In addition, it increases the computational cost considerably depending on the type of curvature produced (biaxial bending).

For this reason, it is advantageous to differentiate the situations where such imperfection does not generate a significant loss of load capacity (Fig. 1(a) for $\nu = \nu_j$), so that a biaxial bending design is not needed, from the situations where it is required (Fig. 1(a) for $\nu = \nu_i$). Only design code BS-8110 [11] proposes an equation to differentiate both situations. This standard indicates that if the geometrical slenderness of the strong axis of the column ($\lambda_{g,\text{strong}} = l_b/h$) is higher than 20, or the ratio "h/b" is equal or higher than 3, a biaxial bending design must be used. If both requisites are not observed simultaneously, a uniaxial bending design with respect to the strong axis is required.

This paper proposes the geometrical slenderness of the weak axis ($\lambda_{g,\text{weak}} = l_b/b$) as the variable to determine whether the influence of such imperfection in the opposite direction to the bending moment is considered or not. This paper terms this boundary value as the "slenderness limit of the weak axis" ($\lambda_{g,\text{weak}}$).

The objective of the paper is to develop an equation to calculate this slenderness limit for non-sway columns, which would be equally valid for rectangular columns with doubly symmetric reinforcement, for normal and high strength concrete, as well as for short-term and sustained loads.

This proposal simplifies the design of columns subjected to bending moments with respect to the strong axis, avoiding biaxial bending design when it is not necessary.

2. Numerical simulation

The procedure for obtaining the equation of the slenderness limit of the weak axis ($\lambda_{g,\text{weak}}$) is described in Section 3. Previously, in this section, a nonlinear numerical simulation using a finite element method for reinforced concrete columns was performed.

This numerical method includes the following main issues:

- 1D finite element with non-constant curvature: the finite element has 13 degrees of freedom (DOFs).
- Nonlinear concrete behavior (Model Code-90 [9], CEB-FIP [12]).
- Nonlinear steel behavior: bilinear diagram (Model Code-90 [9]).
- Geometric nonlinearity: the geometric stiffness matrix and the update of the displacements are included in the definition of the model.
- Time-dependent effects: creep and shrinkage (CEB [13,9]).

A detailed description of the numerical model can be found in [14], where the accuracy degree in the comparison of 468 experimental tests from the bibliography was shown. These tests were on pinned–pinned columns with square or rectangular sections subjected to both uniaxial and biaxial bending but with equal eccentricity at the ends.

The parameters that were studied in these tests and for which the numerical model is valid are the following:

- Relative eccentricity e/h between 0.02 and 1.75.
- Geometric slenderness l/h between 3 and 40.
- Geometrical reinforcement ratio ρ_{rs} between 1% and 5%.
- Steel strength f_y between 298.55 and 684 MPa.
- Height-to-width cross section ratio h/b between 1 and 3.
- Compressive cylinder strength of concrete at 28 days f_c between 10.76 and 104.84 MPa.
- Creep coefficient φ between 0 and 2.90.
- Biaxial bending angle between 0° and 90°.

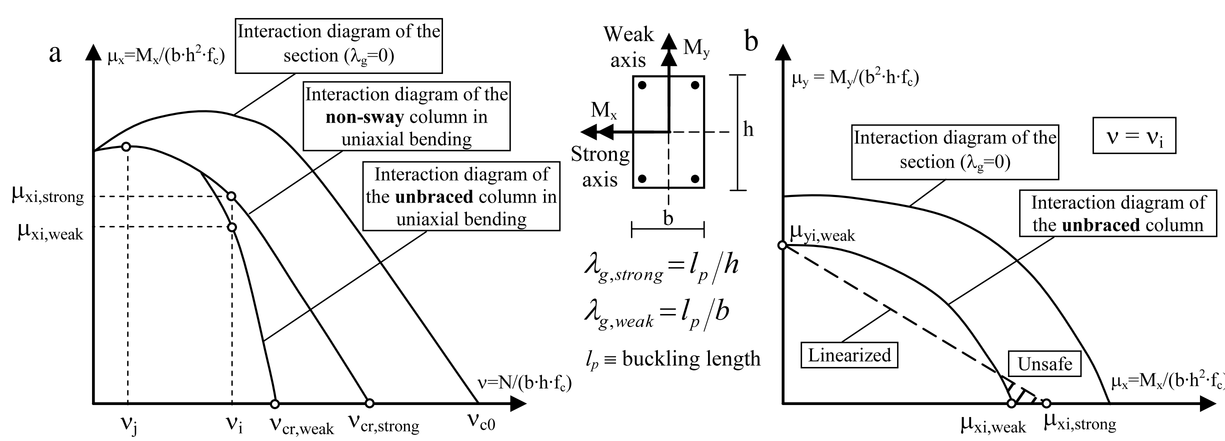

Fig. 1. Dimensionless interaction diagrams. (a) Axial load and bending about the strong axis (ν, μ_x). (b) Biaxial bending (μ_x, μ_y) to a relative axial load (ν_i).