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A differential quadrature method solution for shear-deformable shells of revolution

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Abstract

This paper deals with the application of the differential quadrature method to the linear elastic static analysis of isotropic rotational shells. The governing equations of equilibrium, in terms of stress resultants and couples, are those from Reissner–Mindlin shear deformation shell theory. These equations, written in terms of the circular harmonic amplitudes of the stress resultants, are first put into generalized displacements form by the use of strain–displacement relationships and constitutive equations. The resulting systems are solved by means of the differential quadrature technique with favourable precision, leading to accurate stress patterns.

Keywords: Shell of revolution; Differential quadrature; Static analysis; Numerical method

1. Introduction

Shells of revolution are common structural elements, and they can be found in many fields of engineering technology. Their use spans over different branches of engineering, such as pressure vessels, cooling towers, water tanks, and tires. As is well known, for the various shell geometries presented in the literature, only a few cases of limited applicative importance have a closed-form solution [22,14] and this often needs some further simplifications to be obtained. In the last three decades numerical approaches to shell analysis have become more and more important, in a way that nowadays approximate solutions are the only ones to be studied, making shell analysis a rather important area of computational mechanics [27]. Amongst different numerical techniques for both the statics and dynamics of shells, surely the most widely used one is the finite element method. We recall the displacement-based version [16], the mixed version [19,8,20] and the recent hybrid mixed version [12]. Other methods developed for the problem in argument are the strip method [18], the boundary element method [15], the element-free method [13] and the differential quadrature method [17,21,11,26] in the so-called δ-point version.

The differential quadrature was first introduced [3,4] as a rapid tool for solving linear and nonlinear PDE systems. In the recent past, further developments of this numerical approach have brought a quantity of research results in structural mechanics and in computational mechanics [6] and even in other fields of applied sciences [23,5], due to the great versatility and applicability of the differential quadrature method.

The scope of the present paper is to demonstrate an efficient and accurate application of a version of the differential quadrature approach, for the solution of the static problem of doubly curved shells of revolution. The present work is based on a first-order shear deformation theory (FSDT) for thin shells and hence the solution does not need the δ-point technique [7] which by itself introduces further approximation to the modelling of boundary conditions.

The governing equations of static equilibrium for the shell structure are a set of five bi-dimensional partial differential equations with variable coefficients. They are obtained by means of the Principle of Minimum Total

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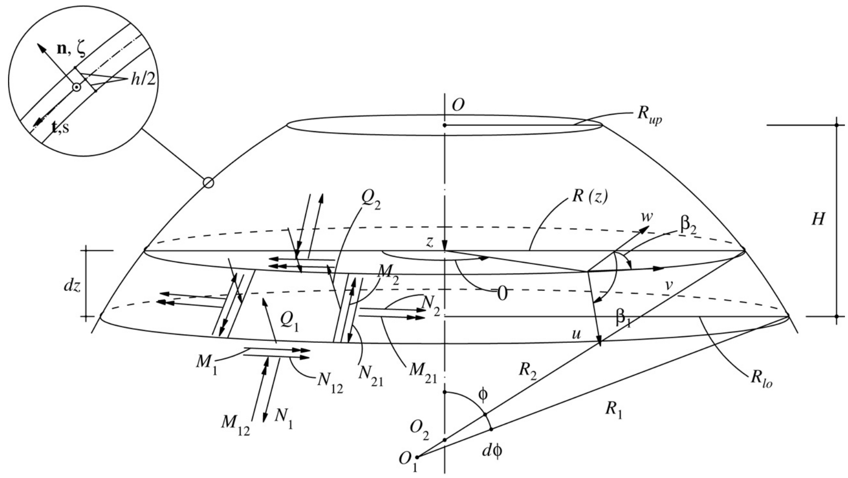

Fig. 1. Shell geometry and coordinate system of the reference middle surface.

Potential Energy for rotational thin shell structures, provided that the strain–displacement relationships for points lying on the reference surface are known. These equations are initially expressed in terms of stress resultants and couples per unit length of parametric lines of the middle surface of the shell.

By introducing the constitutive equations and the kinematic relationships between strain measures and displacements, the equilibrium equations can be written in terms of generalized displacement components only. The expansion of all variables of the problem into Fourier series with respect to the circumferential coordinate θ permits separation of the independent variables, and the initial two-dimensional problem is reduced to a series of simpler one-dimensional problems; in this way, axisymmetric and nonsymmetric loading cases can be easily studied and the resulting governing equations can be solved separately for each circular harmonic component.

Precisely, the governing equations can now be discretized with the aid of the differential quadrature technique so as to give a series of sets of linear algebraic equations, which are solved routinely. The solution obtained so far is given in terms of middle surface translations and rotations. A simple interpolation rule (Lagrange interpolation) along the meridional coordinate direction is sufficient to obtain an accurate pattern for the complete kinematic assessment of the shell. Applying the strain displacements relations to the approximated displacement field and the constitutive equations to the strain measures yields the stress resultants and couples for the shell under consideration. Several examples treated in the present paper show how this simple and efficient procedure produces reliable and accurate results and provides the basis for a further development of the research to the field of more complex shaped rotational shells. The approximate solution shows good convergence characteristics and appears to be accurate when tested by comparison to finite element analyses or analytical solutions available from the scientific literature.

2. Shell geometry and fundamentals

The typical rotational shell with its reference middle surface is presented if Fig. 1. The coordinates on the reference surface are the vertical abscissa z and the circumferential angle θ, while the position of a point lying out of the middle surface is given by the distance ζ, measured along the outward normal to the reference surface. Following this notation and according to Fig. 1, the reference domain of the shell is described by the following bounded open set:

$$\Omega = \{(z,\theta,\zeta) \in \mathbb{R}^3 \mid (z,\theta,\zeta) \in (0,H) \times [0,2\pi] \times (-h/2,+h/2)\}$$

where H is the total height of the shell and h is the thickness; while the boundary of the domain takes the following representation:

$$\partial\Omega = \{(z,\theta,\zeta) \in \mathbb{R}^3 \mid (z,\theta,\zeta) \in (0,H) \times [0,2\pi] \times \{-h/2,+h/2\} \cup \{0,H\} \times [0,2\pi] \times [-h/2,+h/2]\}.$$

As we will be working with an engineering shell theory it is convenient to define the reference domain middle surface or reference surface as

$$\omega = \{(z,\theta,\zeta) \in \mathbb{R}^3 \mid (z,\theta,\zeta) \in (0,H) \times [0,2\pi] \times \{0\}\}$$

and its boundary

$$\partial\omega = \{(z,\theta,\zeta) \in \mathbb{R}^3 \mid (z,\theta,\zeta) \in \{0,H\} \times [0,2\pi] \times \{0\}\}$$

which, in the present case, reduces to the set given by the two extreme parallel circles (Fig. 1).

The shell geometry can be described in different ways: here the parallel radius scalar equation $R = R(z)$ and the thickness variation $h = h(z)$ are given as well as the characteristic parameters $R_{up} = R(0)$ and $R_{lo} = R(H)$, which are respectively the lower and upper parallel radius of the shell corresponding to the boundary parallel circles (Fig. 1).