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Elastic buckling of web plates in I-girders under patch and wheel loading

T. Ren, G.S. Tong*

Department of Civil Engineering, Zhejiang University, Hangzhou 310027, China

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Abstract

This paper makes a brief review of the earlier research on the elastic buckling of rectangular plates with simply supported and clamped boundary conditions, as well as the buckling of web plates in I-girders subjected to patch load. New investigation has been carried out to simulate the realistic load and the restraining conditions of the web plates in I-girders. The buckling of a large number of models under patch load was analyzed with ANSYS and formulae were proposed to predict the elastic buckling coefficients of webs in I-girders. The rotational restraints provided to the web plates by the flanges are considered accurately in the suggested formulae.

The wheel load is another kind of patch load rarely touched in the literature. This paper suggests a simple model to determine the bearing stresses on the top edge of the web plates in I-girders where the effect of the crane rail rigidity is considered. Based on this model, the buckling of the web plate is analyzed and formulae with excellent accuracy are suggested to predict the buckling load.

Keywords: Patch loading; Wheel loading; Web plates in I-girders; Elastic buckling coefficients

1. Introduction

Concentrated forces applied perpendicularly to the upper flanges of I-girders, a load case usually referred to as patch loading, are common in most steel structures, as shown in Fig. 1. Although a lot of concentration has been focused on this problem, the analytic solution has not been obtained due to the complexity of the problem. Therefore, most investigators used the finite element method to study the behavior of the webs under patch loading.

The elastic buckling strength of a rectangular plate under patch loading can be expressed as follows:

$$F_{\mathrm{cr}} = k_{\mathrm{cr}} \frac{\pi^2 E}{12(1-\mu^2)} \frac{t_{\mathrm{w}}^3}{h_{\mathrm{w}}} \tag{1}$$

in which k_{cr} is the elastic buckling coefficient of the plate, h_{w} and t_{w} are height and thickness of the plate, and E and μ are modulus of elasticity and Poisson's ratio of the material, respectively.

Lagerqvist [1] made an excellent review on this topic. Only a brief review is given in the following.

Girkmann in 1936 was the first researcher to study the elastic stability of a rectangular, simply supported plate subjected to a single edge force. His results applies only to plates with aspect ratio, a/h_{w} less than 1.1 and the solution was given in the form of a determinant which had to be evaluated for any particular case. Based on the energy method, Zetlin [2] investigated the elastic buckling of rectangular plates with different aspect ratios under patch loading. It was assumed that the applied load was equilibrated by parabolically distributed shear stresses along the two vertical edges of the plate. White and Cottingham [3] studied this problem using the finite difference method.

Rockey [4] presented an investigation of buckling of simply supported and clamped rectangular plates under partial edge loading using a finite element method. The study was expanded to webs in I-girders. For webs in I-girders, the stresses are assumed to be uniform along the vertical edges and the vertical edges were allowed to rotate as a rigid body about the neutral axis of the section so that the Bernoulli' assumption is valid as in the web of an actual plate girder. After comparing with the

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* Corresponding author. Tel.: +86 571 87952259; fax: +86 571 87961697.
E-mail address: tonggs@zju.edu.cn (G.S. Tong).

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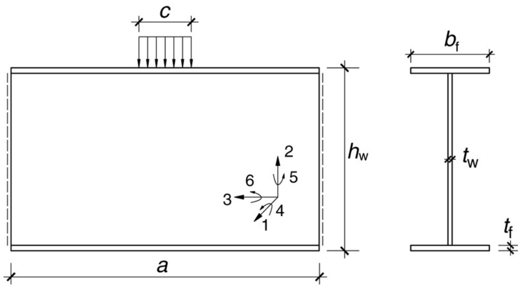
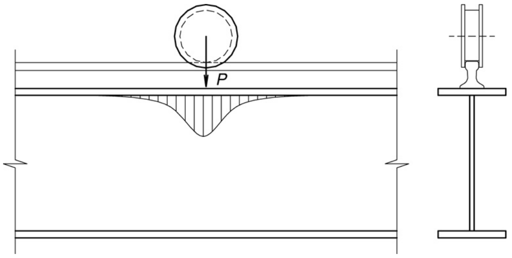


(a) Patch loading.



(b) Wheel loading.

Fig. 1. The model of the I-girder under patch loading and wheel loading.

buckling of the rectangular plate with identical size, it was found that the buckling coefficient of the web increased significantly because of the flange, the buckling mode was also different from that of a rectangular plate. Based on the energy approach suggested by Alfutov and Balabukh, Khan [5–7] presented solutions for the buckling of webs in I-girders. The assumed stress distribution satisfied only the equilibrium conditions, and the corresponding strains may not satisfy the compatibility requirement. Using the same stress distribution as in Khan's theory and the assumed buckling displacement $w = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \sin \frac{i\pi x}{a} \sin \frac{j\pi y}{h_w}$, Robert [8] presented solutions for buckling coefficients of webs in I-girders under patch loading on the basis of the Galerkin method. Moriawaki and Takinoto [9] presented a formula for calculating the resistance for concentrated loads applied at one flange. The formula includes the critical buckling load and the author gave an expression for the buckling coefficients in which the flange thickness was included, therefore the influence of the flanges has been considered to some extent. Graves Smith [10,11] has also concentrated on this problem by using a finite stripe method. Kitipornchai [12] utilized a finite element method using thin-plate elements to investigate the problem studied by Rockey [4] and Khan [5]. The aim of doing this was to validate his finite element formulation, this validated also the results by Rockey and Khan.

Based on a number of numerical analyses, Graciano and Lagerqvist [13] suggested the following equation for the buckling coefficients of webs in I-girders under patch loading.

$$k_{cr} = 5.82 + 2.1 \left(\frac{h_w}{a}\right)^2 + 0.46\sqrt[4]{\beta} \tag{2}$$

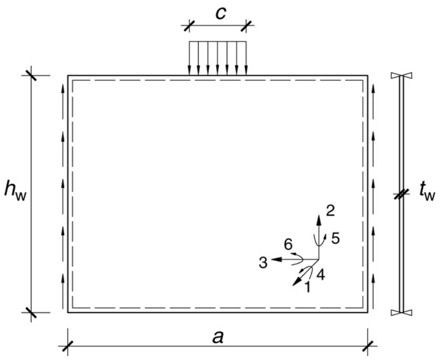


Fig. 2. The model of simply supported rectangular plate.

in which $\beta = \frac{b_f t_f^3}{h_w t_w^3}$ is the factor considering the rotational restraint from flanges. The physical meaning of the factor β is the ratio of the flange free torsional stiffness to the bending stiffness of the web. We could imagine that when β is very small, the rotational restraint from flanges to webs is too slight to restrict the rotation of the web, the web, therefore, behaves like simply supported plates. When β becomes greater, approaching ∞, the web behaves like a clamped plate. Eq. (2) gives an infinite buckling coefficient in this case which obviously violates the requirement of approaching a clamped plate. Therefore, it is worthwhile to further investigate the elastic buckling behavior of the webs in I-girders under patch loading and explain the restraining effects of the flanges properly.

The webs of crane runway girders are subjected to wheel loadings, as shown in Fig. 1(b). The buckling of webs under wheel loading, although similar to the patch loading, is rarely reported in the literature. This paper will also address this problem.

2. Validation of the analysis method

A finite element analysis with the general-purpose finite element package ANSYS was conducted to investigate the elastic buckling behavior of a simply supported and a clamped rectangular plate, as well as the webs in I-girders under patch loading. Uniform loads were applied on the top edge of the plate over a limited length or on the flanges of I-girders. For single rectangular plates, focus was put on the effects of the length–height ratio, a/h_w, on the elastic buckling coefficients, while the effects of the elastic restraints at the flange–web juncture to the elastic buckling coefficients of webs in I-girders were well concerned. The four-node elastic shell element SHELL63 was adopted in ANSYS, it has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes.

Shown in Fig. 2 is the model adopted by Rockey [4], Shahabian and Robert [8] and Graciano and Lagerqvist [13] in their studies. The out-of plane displacements at the four edges are prevented and the vertical displacements at the left