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Experimental identification of rational function coefficients for time-domain flutter analysis

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Abstract

For time-domain flutter analysis, the frequency-dependent self-excited forces acting on the structure can be approximated in the Laplace domain by Rational Functions. The existing Rational Function Approximation (RFA) approach involves approximation of the experimentally obtained flutter derivatives. This motivated the formulation of a system identification technique (Experimental Extraction of Rational Function Coefficients or E2RFC) to directly extract the Rational Function Coefficients from wind tunnel testing. The current formulation requires testing of the section model at lesser number of velocities compared to the flutter-derivative approach which may lead to a significant reduction in time and resources associated with indirect extraction of Rational Functions from flutter derivatives. The methodology and algorithm of the E2RFC method, results of numerical simulation to test the method with two bridge deck sections, and experimentally obtained Rational Function Coefficients for one bridge deck section have been presented.

Keywords: Flutter analysis; Wind tunnel tests; System identification; Rational functions; Bridges

1. Introduction

Aeroelasticity refers to the phenomenon wherein aerodynamic forces and structural motions interact significantly. Flutter is an aeroelastic self-excited oscillation of a structural system. Scanlan and Tomko [1] developed a frequency-domain flutter analysis technique using experimentally obtained flutter derivatives. The resulting equations of motion involve reduced frequency-dependent flutter derivatives whose solution requires an iterative procedure for the determination of critical flutter wind speed. Scanlan [2] and Sarkar et al. [3] were successful in identifying eight flutter derivatives simultaneously from noisy displacement time histories by the modified Ibrahim time domain (MITD) method. Brownjohn and Jakobsen [4] used the covariance block Hankel matrix (CBHM) method for parameter extraction of a two degree-of-freedom (DOF) system. Gu et al. [5] and Zhu et al. [6] used an identification method based on unifying least squares (ULS) theory to extract flutter derivatives of a two-DOF model. Chen et al. [7] used general least-squares theory and Gan Chowdhury and Sarkar [8] used the Iterative Least Squares (ILS) method for identifying eighteen flutter derivatives of three-DOF bridge sections. Time-domain flutter analysis, that uses a frequency *independent* state-space equation to represent the equations of motion, has been gaining popularity in recent times because: (1) flutter analysis does not require iterative calculations while solving the complex eigenvalue problem to determine the frequencies, damping ratios, and mode shapes at different velocities, (2) structural and aerodynamic nonlinearities can be incorporated in the analysis, (3) this formulation can be used to design efficient vibration control systems for suppression of flutter. In time-domain modeling, the frequency dependent aerodynamic self-excited forces acting on the structure can be approximated in the Laplace domain by Rational Functions. The time-domain flutter analysis through Rational Function Approximation (RFA) of the flutter derivatives was done in the aeronautical field by Roger [9] and

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Karpel [10]. The RFA approach is a frequency-independent formulation of unsteady aerodynamic forces in the Laplace domain that involves approximation of the experimentally obtained flutter derivatives through a 'multilevel linear and nonlinear optimization' procedure. Roger [9] formulated the least-squares RFA formulation (LS-RFA) and Karpel [10] developed the minimum state RFA formulation (MS-RFA). Both approaches entail the experimental extraction of flutter derivatives for a section model which represents the span-wise section of the prototype structure under investigation. Flutter derivative extraction involves wind tunnel experiments on the section model at several wind speeds. Only after finding the flutter derivatives can RFA be used for approximating the unsteady self-excited forces; thus it is an indirect approach. RFA formulation has been applied to bridge aerodynamics by several researchers such as Xie [11], Xiang et al. [12], who developed the CVR or Coupled Vibration Record method, and others such as Fujino et al. [13], Wilde et al. [14], Boonyapinyo et al. [15], Wilde et al. [16], Chen et al. [17,18], and Chen et al. [19]. Since RFA is an indirect approach, it motivated the development of a more direct approach for extracting the Rational Function Coefficients from wind-tunnel measurement of aeroelastic forces and displacements rather than approximating the frequency-dependent flutter derivatives. The current system identification technique named Experimental Extraction of Rational Function Coefficients or the E2RFC method is a new approach that uses displacement and surface pressure time histories of a section model subjected to free-vibration tests to identify the unsteady aerodynamic forces and then extract the Rational Function Coefficients. The proposed approach eliminates the need of testing the model through a wide range of wind speeds to identify the flutter derivatives that is required for extracting the Rational Function Coefficients using the RFA. This method requires testing of the model at fewer wind speeds (say two to five) that leads to a significant reduction in time and resources that are associated with the extraction of flutter derivatives and use of that data to get RFA. Recording dynamic surface pressures on wind tunnel models has become a routine job with the introduction of transducers that are capable of high sampling rates (>200 Hz per channel). The accuracy of these transducers has improved significantly with digital sensors. Further, the forced-vibration method for identifying flutter derivatives already requires measurement of aeroelastic forces either directly or indirectly using surface pressures. Therefore, the proposed method does not necessarily increase the complexity of the experimental methods commonly applied to section model tests.

The current paper focuses on the new system identification method developed for direct extraction of Rational Function Coefficients. Numerical simulation results are presented to validate the effectiveness of the method using a streamlined deck section (B1) and a relatively bluff deck section (a deep truss) of the Akashi Kaikyo bridge in Japan (B2). The experimental set-up used for Rational Function Coefficient extraction has been described. Finally, experimentally obtained Rational Function Coefficients are presented for a streamlined bridge deck section (B3) that is different from B1.

2. Current system identification method

2.1. Equations of motion

In the current formulation, a section model is assumed to have two degrees of freedom: the vertical deflection h of the local center of gravity (c.g.), and the rotation α about that c.g. Also, m and I_α are the mass and the mass moment of inertia per unit length of the sectional model, respectively. c_h, c_α, are the damping and k_h, k_α are the stiffness coefficients of the heaving and pitching modes, respectively. The equations of motion for the section model in a smooth flow subjected to aeroelastic lift (L_{ae}) and moment (M_{ae}) can be written as:

$$m\ddot{h} + c_h\dot{h} + k_h h = -L_{\mathrm{ae}} \tag{1}$$

$$I_\alpha\ddot{\alpha} + c_\alpha\dot{\alpha} + k_\alpha\alpha = M_{\mathrm{ae}}. \tag{2}$$

The aeroelastic lift and moment can be written as follows:

$$L_{\mathrm{ae}} = 0.5\rho U^2 B\left[KH_1^*\frac{\dot{h}}{U} + KH_2^*\frac{B\dot{\alpha}}{U} + K^2H_3^*\alpha + K^2H_4^*\frac{h}{B}\right] \tag{3}$$

$$M_{\mathrm{ae}} = 0.5\rho U^2 B^2\left[KA_1^*\frac{\dot{h}}{U} + KA_2^*\frac{B\dot{\alpha}}{U} + K^2A_3^*\alpha + K^2A_4^*\frac{h}{B}\right] \tag{4}$$

where ρ is the air density, U is the mean cross wind velocity, B is the width of the section model, $K = B\omega/U$ is the non-dimensional reduced frequency, ω is the circular frequency of oscillation. The non-dimensional aerodynamic coefficients H_i^* and A_i^* $(i = 1, \ldots, 4)$ are the flutter derivatives and they evolve as functions of the reduced velocity U/fB (where $f = \omega/2\pi$ is the frequency of oscillation). These coefficients can be identified using frequency-domain methods mentioned earlier.

Frequency-dependent aeroelastic forces are often transformed into time-dependent forces so that they can be applied in the explicit time-domain approach. The most common form of the approximating function for the aeroelastic force coefficients, used in aeronautics, is a Rational Function of the non-dimensional Laplace variable p (non-dimensional Laplace variable, $p = sB/U = iK$, where non-dimensional time, $s = Ut/B$). For the two degree-of-freedom (DOF) sectional model, the equations of motion considering aeroelastic forces only, can be written in the Laplace domain (L denotes the Laplace operator) with zero initial conditions as:

$$(\underline{M}p^2(U/B)^2 + \underline{C}p(U/B) + \underline{K})L(\underline{q}) = \underline{V}_f\,\underline{\tilde{Q}}L(\underline{q}) = \underline{V}_f\,\underline{\tilde{Q}\hat{q}} \tag{5}$$