Available online at www.sciencedirect.com
SCIENCE DIRECT®
Engineering Structures 27 (2005) 629–638
ENGINEERING STRUCTURES
www.elsevier.com/locate/engstruct

# On the symmetries and vibration modes of layered space grids

Alphose Zingoni*

*Department of Civil Engineering, University of Cape Town, Rondebosch 7701, Cape Town, South Africa*

Received 1 April 2004; received in revised form 14 October 2004; accepted 8 December 2004

**Abstract**

Layered space grids (typically double-layer and triple-layer configurations) offer large-span roofing solutions for many modern facilities ranging from industrial warehouses to exhibition pavilions, places of public assembly and indoor sports complexes. These configurations often exhibit a high degree of symmetry not only at the modular level, but also at the global level, and the vibration characteristics of the structures are heavily influenced by the type of symmetries they possess. In this contribution, the symmetries of various configurations of layered space grids are described, and the associated vibration modes are explored using a group-theoretic approach. This allows insight to be gained, prior to any detailed analysis, on the number of distinct modes of vibration (and natural frequencies) of specific symmetry types, and some important observations are made in this regard. Computational aspects are also covered, and a numerical example included.


*Keywords:* Layered space grids; Vibration analysis; Symmetry modes; Group theory; Symmetry groups

## 1. Introduction

Layered space grids, on account of their high stiffness and strength-to-weight ratio, find application as large-span roofing solutions for a variety of facilities such as industrial warehouses, exhibition areas, airport concourses, shopping malls, and indoor sports arenas. The structures are characterized by a large number of identical repeating units, which makes their assembly particularly economical. If the overall arrangement is symmetric, as is quite often the case, then advantage can be taken of this, to gain some important insights into the vibration characteristics of the grid, and to considerably simplify the vibration analysis itself.

To facilitate the complete description of the spatial configuration of skeletal space structures such as layered grids, a number of schemes has been proposed by various investigators, and notable among these is Formex configuration processing [1], developed at the University of Surrey in the UK. Such schemes greatly simplify data handling for structures of the type in question, by exploiting the repetitiveness (translational and cyclic) inherent in the configurations.

Owing to the symmetry inherent in lattice domes and related space structures, group theory has been employed by a number of investigators [2–4], to study the bifurcation behaviour of these structures. Domes belonging to the dihedral groups (i.e. groups describing the symmetry of regular polygons) are very common, and several studies have focused on these. For instance, Healey [2] studied the global bifurcation problem of a lattice dome with hexagonal symmetry, and constructed a reduced problem having fewer unknowns than the original problem, where solutions of the reduced problem are exact solutions of the full problem. Group-theoretic techniques, apart from simplifying the numerical computation of a problem, may be used to extract useful qualitative information on the problem in advance of the computations. Ikeda et al. [3] deduced, on this basis, a hierarchy for the symmetry-breaking process of bifurcation for a configuration with dihedral symmetry.

A computational scheme combining group-theoretic ideas and sub-structuring techniques has been proposed by Healey and Treacy [5], for tackling vibration eigenvalue problems of skeletal structures with symmetry. Block-diagonalisation of the matrices was achieved not on the basis of transforming the global mass and stiffness matrices for the full structure, but on the basis of the matrices for

* Tel.: +27 21 650 2601; fax: +27 21 689 7471.
*E-mail address:* azingon@eng.uct.ac.za.

0141-0296/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.engstruct.2004.12.004

the repeating sub-structure, a procedure particularly advantageous in the case of large-scale problems. Zlokovic [6] employed group theory to simply problems of the static, stability and vibration analysis of various types of structures (beams, frames, taut strings and grillages), while Zingoni [7] employed similar techniques to tackle the specific problem of the vibration analysis of high-tension cable nets with rectangular and square symmetries. In all cases, the vector space of the normal variables of the problem is decomposed into a number of independent subspaces spanned by symmetry-adapted variables, permitting the system of equations for each subspace (with only a fraction of the number of unknowns in the original problem) to be solved for separately. The overall computational effort is substantially reduced, in comparison with conventional methods of analysis. A general review of applications of group theory to problems in solid and structural mechanics has recently been published [8].

In this study, we focus on a variety of configurations for layered space grids commonly adopted as roofing solutions, and begin by identifying their symmetry elements and symmetry groups. We are interested in studying the vibration characteristics of the grids under small transverse motions. For this purpose, a lumped-parameter model is adopted, with the space grid members assumed to possess elasticity but no mass, and the mass of the grid assumed to be concentrated at the joints or nodes. For structures of the type in question, such a lumped-parameter model is quite realistic and reasonably justified, since the 3-dimensional convergence of a relatively high number of members at a joint has the effect of concentrating the mass of the system around the joints, an effect further enhanced by the concentrated actual mass of the connector units (often consisting of solid metal spheres) that occur at the nodes. We then make use of group theory to predict the number of vibration modes of a given symmetry type, which allows considerable insight to be gained on the vibration characteristics of the structure, even before the computations are undertaken. The computational procedure for extracting natural frequencies of vibration and actual mode shapes for the various configurations is also indicated, and a numerical example included.

## 2. Symmetries of layered space grids

Fig. 1 depicts layouts in plan and elevation of some typical symmetric configurations of layered space grids. For each of the four basic layouts, the uppermost diagram shows the plan view of the grid, with nodal positions in plan and axes of symmetry numbered or labelled as shown. The lower two diagrams are elevations of the grid, the upper of these depicting a *double-layer grid* supported at the bottom corner nodes, and the lower depicting a *triple-layer grid* supported at the corner nodes of the middle-layer. In these elevations, the $z$ axis points in the upward vertical direction, while the $h$ axis denotes the (horizontal) plane of all the horizontal axes appearing in the plan view. Members, joints and supports all conform, with respect to size and type, to the overall symmetry of the configuration in question and, additionally, the triple-layer grids are symmetric about the middle layer. These assumptions are consistent with arrangements of such configurations in practice.

The triangular grid of Fig. 1(a) has, for the double-layer configuration, 10 nodes in the lower layer (3 of which are supported and not numbered in the diagram) and 6 nodes in the upper layer (whose positions in plan coincide with the centroids of 6 of the triangular panels of the lower layer), giving a total of 13 degrees of freedom corresponding to the small vertical motions of the 13 unsupported nodes. The total number of members making up the grid is 45 (i.e. 18 in the lower layer, 9 in the upper layer, and 18 linking the two layers). For the triple-layer grid, we have 10 nodes in the middle layer (3 of which are supported and not numbered) and 6 nodes in each of the top and bottom layers, giving a total of 19 degrees of freedom and 72 members.

For the triangular grid, the double-layer configuration has 3-fold rotational symmetry about the central vertical $z$ axis (which passes through Node 4) and 3 reflection planes, giving it 6 symmetry elements:

$$\{e, C_3, C_3^{-1}, \sigma_1, \sigma_2, \sigma_3\}$$

where $e$ is the identity element, $C_n$ and $C_n^{-1}$ clockwise and anticlockwise rotations, respectively, of $2\pi/n$ about the central $z$ axis, and $\sigma_i$ $(i = 1, 2, 3)$ a reflection in the vertical plane containing the $i$ axis. The double-layer configuration thus belongs to the symmetry group $C_{3v}$.

In addition to the above symmetry elements, the triple-layer configuration (lowest diagram of Fig. 1(a)) has horizontal symmetries and rotary reflections. The full set of symmetry elements has 12 members, namely:

$$\{e, C_3, C_3^{-1}, \sigma_1, \sigma_2, \sigma_3, C_2^1, C_2^2, C_2^3, \sigma_h, S_3, S_3^{-1}\}$$

where $C_2^i$ $(i = 1, 2, 3)$ is a rotation of $\pi$ about the horizontal $i$ axis, $\sigma_h$ a reflection in the horizontal $h$ plane of the middle layer, and $\{S_n, S_n^{-1}\}$ are clockwise and anticlockwise rotary-reflections of $2\pi/n$ (rotations of $2\pi/n$ about the central vertical $z$ axis, followed by a reflection in the horizontal $h$ plane. The triple-layer configuration of Fig. 1(a) therefore belongs to the symmetry group $D_{3h}$.

The hexagonal grid of Fig. 1(b) has 19 nodes in the lower layer (in the case of the double-layer grid), or the middle layer (in the case of the triple-layer grid), 6 of which are supported (un-numbered corner nodes in the plan view). Each of the top and bottom layers of the triple-layer grid, or just the upper layer in the case of the double-layer grid, has 24 nodes whose positions in plan coincide with the centroids of the 24 triangular panels of the middle layer of the triple-layer grid or of the lower layer of the double-layer grid. There are therefore 37 degrees of freedom for the double-layer grid and 61 degrees of freedom for the triple-layer grid, corresponding to the small vertical motions of the unsupported nodes. The total number of members may easily