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Mechanical behaviour and non-linear analysis of short beams using softened truss and direct strut & tie models

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Abstract

This paper discusses the application of the softened truss and strut and tie models on short beams. The softened truss model originally proposed by Hsu is based on three fundamental principles of mechanics of materials, i.e., stress equilibrium, strain compatibility, and constitutive laws of materials. The model has two important characteristics. The first is the non-linear association of stress and strain. The second is the softening of concrete in compression due to tensile strains in the perpendicular direction. For short beams, one of the most important factors influencing behaviour is the effective transverse compression, which is dependent on the shear-span-to-depth (a/d) ratio. In this study, the softened truss model is revised so that the model gives more accurate predictions of the shear strength of short beams. The revised softened truss model (RSOTM) has three differences from the original model. In the RSOTM, it is shown that the effective transverse compression is not only dependent on the a/d ratio but also on the reinforcement ratio in the longitudinal direction as well as the size of the specimen. Second, it is shown that the softening phenomenon in short beams is more severe than that assumed in the original model. The third difference is that the effective transverse compression will be zero when a/d ratio exceeds 1.5. The ROSTM is compared with ACI318-2002, which recommends that short and deep beams should be designed by strut and tie models. ACI318-2002 and the final draft of Eurocode 2 do not provide specific guidance on suitable strut and tie models for different cases. In this study on short beams, a strut and tie model which consists of three mechanisms is recommended: a direct strut mechanism to account for the contribution of concrete and two truss mechanisms to account for the contributions of the horizontal and vertical shear reinforcements to the shear strength. It is imperative that ACI 318-2002 and the final draft of Eurocode 2 should be modified so that they should stress the existence of these three mechanisms in short and deep beams.

Keywords: Shear strength; Mechanics of materials; Non-linear analysis; Non-linear programming; Deep beams; Span–depth ratio; Reinforced concrete; Strut and tie models; Building codes; Diagonal tension; High-strength concrete; Structural analysis; Size effect; Stress–strain relationships; Shear properties; Deformation; Trusses; Equilibrium; Constitutive laws

1. Introduction

Short beams are very frequently used in floor diaphragms, shear walls, transfer girders used in multi-storey buildings, in foundation walls and in folded plate roof structures. ACI ASCE Committee 445 [5] classifies a beam with shear span-to-depth ratio, a/d, less than 1 as a deep beam, and a beam with a/d exceeding 2.5 as an ordinary shallow beam. Any beam between these limits is classified as a short beam. The typical stress condition in a web of deep beams is shown in Fig. 1.

Hsu et al. [1–6] have made very important and valuable contributions to the shear analysis and mechanical behaviour of reinforced concrete structures. The softened truss model was first proposed at the University of Houston by Hsu [1–5] for predicting not only the strengths but also the deformations of structures in the non-linear analysis of short beams. The model has been applied on 64 deep and short beam tests by Mau and Hsu [6]. However, the database the model was applied on did not contain tests with high strength concrete or tests with greater size or tests with a

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Notations

α	is the angle of inclination between the longitudinal axis and the d axis
β_s	is the effectiveness factor for the strut
ε_1	is the principal tensile strain in the concrete
ε_r and ε_d	are the average normal strains in the r and d directions, respectively
ε_{ly} and ε_{ty}	are the yield strains of longitudinal and transverse steel bars respectively
ε_o	is the strain at the maximum compressive stress of non-softened concrete and can be taken as (-0.002)
ε_{cr}	is the cracking tensile strain
ε_l and ε_t	are the average normal strains in the l and t directions, respectively
σ_l and σ_t	are the average concrete stresses in the l and t directions respectively
ϕ	is a strength reduction factor which is equal to 0.75 for all truss elements
ϕ_{STM}	is the value of ϕ for struts, ties, and nodal zones in strut and tie models
γ	is the angle from axis of strut and axis of a layer of confining reinforcement
θ	is the angle between the axis of the compressive strut and the axis of the tie attached to one end of the strut
γ_{lt}	is the average shear strain in the l–t co-ordinate
ρ_l and ρ_t	are the steel ratios in the l and t directions, respectively
γ	is the angle from axis of strut and axis of a layer of confining reinforcement
θ	is the angle between the axis of the compressive strut and the axis of the tie attached to one end of the strut
ρ	is the main tensile reinforcement ratio $=100As/bd$
ρ_l	is the longitudinal reinforcement ratio including the horizontal shear reinforcement
ρ_t	is the vertical shear reinforcement ratio
τ_{lt}	is the shear stress
ν	is the effectiveness factor
a	is the shear span
A_v	is the area of the vertical shear reinforcement and A_h is the area of the horizontal shear reinforcement
A_v	is the area of shear reinforcement perpendicular to the main flexural steel within a distance s in ACI Code
A_{vh}	is the area of the shear reinforcement parallel to the main flexural steel within a distance of s_2 in ACI Code
b_w	is the web width of T beams in inches
b	is the width of the rectangular beams
d	is the depth of the beam cross-section
E_s	is the modulus of elasticity of steel bars
f_c	is the concrete cylinder strength in MPa
f_c	is the maximum compressive cylinder strength
f_{cr}	is the concrete cracking stress taken as $0.31\sqrt{f_c}$ (where f_c is in MPa) in SI units
f_{cu}	is the effective compressive strength of the concrete in a strut
f_l and f_t	are the average stresses in longitudinal and transverse steel bars, respectively
f_s	is the average stress in the beam reinforcement
f_{yl}	is the yield strength of the reinforcement
f_{ty}, f_{ly}	are the yield stresses of longitudinal and transverse steel bars, respectively
f_{wh}	is the average stress in the transverse reinforcement in the horizontal direction
l_b	is the length of the bearing plate
l_e	is the effective span as measured from center-to-center of support points
h	is the cross-sectional height of the beam
F_u	is the force in the strut or tie, or force acting on a node due to the factored loads
M_u	is the moment, at factored loads, occurring at the critical section
N	is the number of specimens
SD	is the standard deviation
s_h	is the spacing of the horizontal, s_v the vertical shear reinforcement
x_e	is the clear shear span measured from inside edge of the bearing block at support to outside edge of the bearing block at loading point
V	is the shear force in N
V_c	is the contribution of the concrete and V_s is the contribution of the web steel if present to the shear strength
$V_{\exp}$	is the measured ultimate strength of deep beams
V_n	is the calculated shear strength
w	is the width of the diagonal concrete strut mechanism, w_{req} is the required width of the struts or nodal zones
w_s	is the width of the strut
w_t	is the effective width of the tie

significant amount of shear reinforcement. Furthermore, the database that Mau and Hsu applied their model on contained both deep beam (a/d less than 1) and short beam (a/d greater than 1) tests. The database also contained tests carried out before the 1980s. In this study on the other hand, a new database consisting of a greater number of specimens is used to investigate the predictions of the softened truss model. The tests belonged to different researchers such as