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Modified sliding mode control using a target derivative of the Lyapunov function

Sang-Hyun Lee[a], Kyung-Won Min[b,*], Young-Cheol Lee[c]

[a] *Research Institute of Engineering Science, Seoul National University, Seoul, Republic of Korea*
[b] *Department of Architectural Engineering, Dankook University, Republic of Korea*
[c] *CHANG & MINWOO Structural Consultant Co., Ltd., Seoul, Republic of Korea*

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Abstract

This paper presents a modified sliding mode control (SMC) algorithm for vibration control of structures, designed to enhance the control performance of the widely used SMC algorithm. In the modified SMC, the control force is determined so as to meet conditions imposed on the target derivative of the Lyapunov function. A shape function is developed to determine which one of the equivalent and corrective control—which are the two terms comprising the SMC—is the dominant part in controlling structures. Simulation results for linear and nonlinear systems show that the proposed method is able to enhance the control performance of the original SMC. Also, the modified SMC can be applied not only to active control, but also to semi-active control which is suitable for practical application. Moreover, it is observed that the performance is insensitive to the fundamental vibrating period and utilizes less control energy as compared to the original SMC.
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1. Introduction

During the last few decades, significant efforts have been made to develop active control devices and algorithms for large scale civil structures subjected to earthquake loads, and the effectiveness has been verified through extensive analytical and experimental studies [1–4].

For the practical application of an active control strategy to civil engineering, the problem of stability and robustness is one of the major issues and this is examined in [5,6]. Design of a stable and robust controller is possible using Lyapunov stability theory, which requires the definition of a positive definite Lyapunov function, and the corresponding controller is designed so as to make the derivative of the Lyapunov function negative semi-definite [7–9]. Wu and Soong proposed modified bang–bang control by using Lyapunov's direct method [10]. Dyke et al. used a magnetorheological (MR) damper designed to dissipate energy maximally, choosing as the Lyapunov function the total vibratory energy [11]. Min et al. proposed a probabilistic control algorithm, which determines the direction of a control force by the Lyapunov controller design method [12]. Also, in seismic engineering, it is essential that the controller should have the potential to be effective in controlling a system with nonlinearities such as permanent deformation and hysteresis for multiple earthquakes [13,14].

Sliding mode control (SMC), one of the Lyapunov controllers, is a switching control method [15], and SMC has been applied in the control of civil engineering structures under earthquake and wind loads; its effectiveness and robustness were verified through theoretical and experimental studies on linear or nonlinear systems [16–20]. Also, SMC can be applied to determine the sliding surface where the motion of a structure is stable, and the Lyapunov function is defined as a scalar function proportional to the distance of states from the sliding surface.

* Corresponding author.
E-mail address: kwmin@dankook.ac.kr (K.-W. Min).

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In the SMC framework, the control force is given as the sum of a *corrective control force* and an *equivalent control force* [21]. The corrective control force makes the response trajectory deviating from the sliding surface back into the sliding surface, while the equivalent control force causes the response to be parallel to the sliding surface or, in special cases, keeps the trajectory within the sliding surface. The effectiveness and robustness of SMC depend on which one of the above two forces is the dominant part of the control force, and the effect is strongly related to the dynamic characteristics of the sliding surface determined by the LQR method [22].

According to the control objectives and capacity of the actuator, a sliding mode controller can be a linear one, which generates control force proportional to states or the excitation signal, or a nonlinear one, such as a bang–bang controller, which generates maximum force irrespective of the magnitude of the states and/or excitation. However, since SMC is generally designed so as to satisfy the condition that the derivative of the Lyapunov function is just negative semi-definite, linear controllers derived in previous studies cannot make the most of the actuator and bang–bang controllers generate an unnecessarily large control force.

In this paper, the concept of a target derivative of the Lyapunov function is proposed, for determining the weighting between corrective and equivalent control parts. A shape function is developed for this purpose. This function plays a role similar to that of the saturation function of Lee et al. [23] or the shifted sigmoid function of Ertugrul and Kaynak [24], which are developed to eliminate the chattering which happens in a Lyapunov controller such as a SMC one. Numerical simulations using linear and nonlinear systems under seismic excitations have been performed to evaluate the effectiveness of the proposed algorithm.

2. Design of the sliding mode control

2.1. The equation of motion

The state-space form of the equation for an n-DOF second-order mass–damping–spring system subjected to a ground acceleration $\ddot{x}_g$ and a control force vector $\mathbf{u}$ of size $r \times 1$ is given by

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B_1}\ddot{x}_g + \mathbf{B_2}\mathbf{u} \tag{1}$$

where

$$\mathbf{z} = \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix}, \ \mathbf{A} = \begin{bmatrix} \mathbf{O} & \mathbf{I} \\ -\mathbf{M^{-1}K} & -\mathbf{M^{-1}C} \end{bmatrix}, \ \mathbf{B_1} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M^{-1}E} \end{bmatrix}, \ \mathbf{B_2} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M^{-1}H} \end{bmatrix} \tag{2}$$

and $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are, respectively, the mass, damping, and stiffness matrices of size $n \times n$, r is the number of controllers, $\mathbf{x}$ is the displacement response vector of size $n \times 1$, and $\mathbf{E}$ and $\mathbf{H}$ are the earthquake influence and control force influence matrices, respectively. $\mathbf{I}$ and $\mathbf{0}$ are the identity and zero matrices, respectively.

2.2. Design of the sliding surface

A sliding surface is given as a linear function of the state vector $\mathbf{z}$ such that

$$\mathbf{s} = \mathbf{P}\mathbf{z} \tag{3}$$

in which $\mathbf{s}$ is an r-vector. The matrix $\mathbf{P}$ $(r \times 2n)$ can be determined by the LQR method to minimize the following performance index [16]:

$$J = \int_0^\infty \mathbf{z^T Q z}\, \mathrm{d}t \tag{4}$$

where $\mathbf{Q}$ is a $(2n \times 2n)$ positive definite weighting matrix.

2.3. Control forces

A Lyapunov function V is selected as follows:

$$V(\mathbf{s}) = 0.5\, \mathbf{s^T s}. \tag{5}$$

The derivative of the Lyapunov function is given as follows:

$$\dot{V}(\mathbf{s}) = \boldsymbol{\lambda}(\mathbf{u} - \mathbf{u_{eq}}) = \sum_{i=1}^{r} \dot{V}_i = \sum_{i=1}^{r} \lambda_i (u_i - u_{\mathrm{eq}i}) \tag{6}$$

where

$$\boldsymbol{\lambda} = \mathbf{s}^{\mathrm{T}}\mathbf{P}\mathbf{B_2} = [\lambda_1, \lambda_2, \ldots \lambda_r] \tag{7}$$

$$\mathbf{u}_{\mathrm{eq}} = -(\mathbf{PB_2})^{-1}\mathbf{PAz} - (\mathbf{PB_2})^{-1}\mathbf{B_1}\ddot{x}_g. \tag{8}$$

The control force for $\dot{V}(\mathbf{s}) \le 0$ is expressed as the sum of the equivalent control force $\mathbf{u}_{\mathrm{eq}}$ and the corrective control force $\mathbf{u}_{\mathrm{c}}$, such that

$$\mathbf{u} = \mathbf{u}_{\mathrm{eq}} + \mathbf{u}_{\mathrm{c}} \tag{9}$$

in which $\mathbf{u}_{\mathrm{c}}$ is determined so as to satisfy the condition $\mathrm{sgn}(\boldsymbol{\lambda} \cdot \mathbf{u}_{\mathrm{c}}) \le 0$. $\mathrm{sgn}(\cdot)$ is the signum function.

Eqs. (6) and (9) indicate that $\mathbf{u}_{\mathrm{eq}}$ makes $\dot{V}(\mathbf{s})$ zero and $\mathbf{u}_{\mathrm{c}}$ makes $\dot{V}(\mathbf{s})$ negative.

If a control force is not applied, $\dot{V}(\mathbf{s})$ becomes

$$\dot{V}(\mathbf{s}) = -\boldsymbol{\lambda}\mathbf{u_{eq}} = \sum_{i=1}^{r} \dot{V}_{in} = -\sum_{i=1}^{r} \lambda_i u_{\mathrm{eq}i} \tag{10}$$

where subscript 'i' means the ith controller. It is noted that if $\dot{V}_{in} < 0$, the response trajectory approaches the sliding surface without the help of the controller, which is reasonable since general civil engineering structures show stable behaviors without any controllers. When a designer hopes to realize an asymptotic stability using a controller, i.e. $\dot{V}(\mathbf{s}) \le 0$ at every instant, the following continuous sliding mode controller (CSMC) can be designed [16]:

$$\mathrm{CSMC}: u_i^* = u_{\mathrm{eq}i} - \delta_i \lambda_i \tag{11}$$

in which $\delta_i \ge 0$, and $-\delta_i\lambda_i$ is the corrective force. The control force of Eq. (11) is a linear one, and the

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