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Dynamic instability of a wheel moving on a discretely supported infinite rail

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ABSTRACT

The parametric instability of a wheel moving on a discretely supported rail is discussed. To achieve this, an analysis method is developed for a quasi-steady-state problem which can represent an exponential growth of oscillation. The temporal Fourier transform of the rail motion is expanded by a Fourier series with respect to the longitudinal coordinate, and then the response of the rail deflection due to a quasi-harmonic moving load is derived. The wheel/track interaction is formulated by the aid of this function and reduced to an infinite system of linear equations for the Fourier coefficients of the contact force. The critical velocities between the stable and unstable states are calculated based on the nontrivial condition of the homogeneous matrix equation. Through these analyses the influences of the modeling of rail and rail support on the unstable speed range are examined. Moreover, not only the first instability zone but also other zones are evaluated.

1. Introduction

A railway track consists of rails and equally spaced sleepers. Due to this sleeper distribution, the track is given by the repetition of a representative unit structure. Therefore, the dynamic behavior of this structure is governed by its periodicity. In general, a periodic structure has frequency ranges called stop bands or band gaps in which any wave propagations are forbidden, while propagating wave modes exist in pass bands locating between these frequencies. Since the band analysis is essential to understand the dynamic nature of periodic systems such as railway tracks, the dispersion analyses of infinite beams resting on periodic supports have been made by many researchers [1–3]. While these works are not necessarily focusing on the railway tracks, Thompson [4] exclusively investigated the wave propagation in a rail considering the cross-sectional deformation. However, since in this work the rail support was modeled by continuous layers equivalent to a sub-structure composed of pads, sleepers and ballast, the pinned–pinned resonance cannot be simulated. Abe et al. [5] developed a track model consisting of an infinite Timoshenko beam and spring–mass–spring supports in which the rail in a unit cell given by a sleeper bay was discretized by beam elements. Shimizu et al. [6] considered axially stressed rails and represented a track unit composed of the left and right rails and a sleeper by 3-D beam elements. Through dispersion analyses, they investigated the influence of the axial load on the band structure of the 3-D track model.

Since the track vibration is induced by the varying contact force between a moving wheel and a rail, the dynamic response of a track subjected to a moving load is also a fundamental feature for the assessment of vibration and noise.

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Belotserkovskiy [7] derived a steady-state solution caused by a harmonic load moving on an infinite Euler–Bernoulli beam periodically supported by springs. The developed method is extended to a harmonic load not synchronizing with the sleeper-passing frequency. Taking into account the phase change in the response of a periodic structure, the problem can be reduced to the rail deflection in a sleeper bay. Sheng et al. [8] also obtained a steady-state solution for a periodically supported rail excited by a harmonic load moving with a constant velocity. In order to simulate the rail cross-sectional deformation, the rail section is represented by an assembly of Timoshenko beams for lateral vibration within the framework of the 2.5-D modeling. It was noted that, even for a harmonic load, since the steady-state response due to a moving force is composed of various frequencies, the wheel/track steady-state interaction analysis increases the difficulties. Metrikine [9] developed an analysis method for a track consisting of an infinite rail and discrete sleepers resting on a semi-infinite layered viscoelastic medium. In this work a steady-state solution of the structure subjected to a moving constant load is formulated by the way of the temporal Fourier transform, under the periodicity condition. In this description the steady-state solution does not require any moving coordinate.

The track vibration is caused by the roughness of the rail and wheel surfaces and the varying track stiffness along a sleeper bay due to the discrete support. In general, the former affects over a wide frequency range, in particular governs a predominant behavior around the pinned–pinned resonance frequency. On the other hand, the latter strongly depends on the wheel speed. When the sleeper-passing frequency coincides with that of the wheel/track interaction resonance, a large vibration called parametric resonance can be excited. A number of researchers studied the influences of this effect on the track response. Nordborg [10] found that a harmonic rail irregularity having the same wavelength as the sleeper spacing makes the parametric excitation dominates the dynamic behavior. In this work it was also shown that the stiffness reduction in pads and ballast can contribute to the suppression of the vibration level except around the resonant frequency, and the shortening of sleeper spacing and the enlargement of rail sectional area can be effective remedies for the parametric excitation. In Ref. [11] Nordborg investigated the influence of the stochasticity in the sleeper spacing and pad stiffness on the vibration level. It was concluded that the randomness increases the vibration level irrespective of frequency and thus makes the parametric resonance unobservable. Kruse and Popp [12] performed the steady-state interaction analysis for wheels moving with a constant speed on a track. They claimed that the variation of the wheel/rail contact force is at most 10 percent of the static load, and consequently the contact stiffness can be approximated by a linear spring. Wu and Thompson [13] showed that, although the parametric excitation cannot be comparable to the effect of surface irregularity, it increases with the wheel speed. They also found that the wheel/rail contact force is dominated by the frequencies synchronizing with the passing frequency and by the pinned–pinned resonant frequency. Wu and Thompson [14] estimated the noise induced by the parametric excitation. It was shown that the roughness on the surface dominates the noise at lower speeds. In contrast to this, at high speeds, the noise due to the parametric excitation is no longer negligible. Sheng et al. [15] developed a steady-state interaction analysis method for a series of wheels. In this approach a sinusoidal surface irregularity with wavelength of a multiple of sleeper span is allowed. An approach to obtain a moving irregularity equivalent to the parametric excitation was also proposed. Furthermore, it was found that while the wheel contact forces of the same bogie are indistinguishable, the interaction between the front and rear bogies is significant. Mazilu [16] considered a harmonic roughness between the wheel and rail, and investigated the influences of the wheel mass and velocity on the parametric excitation. Although the coincidence of the sleeper-passing frequency with the wheel/track resonant frequency results in the parametric resonance, the 1/2 and 1/3 sub-harmonic resonances can also be induced at lower velocities as shown in Ref. [17].

Damping in a railway structure limits the parametric resonance only to the sleeper-passing frequency. Therefore, in the above papers the parametric excitation was exclusively discussed based on an assumption that the steady-state is dominated by the sleeper-passing frequency. However, in the case of undamped or lightly damped system, when the passing-frequency of two sleeper spans coincides with the wheel/track resonant frequency, a steady-state solution characterized by twice the sleeper passing period can be excited and instability may be induced. Such phenomenon closely relates to the railway engineering. For example, the parametric instability in the interaction of a pantograph and a catenary wire system is well known [18]. The parametric resonance for a single railway wheelset was discussed by Szabó and Lóránt [19]. From the viewpoint of vehicle/highway bridge interaction, Chung and Genin [20] analyzed the stability of a two mass system moving on a multispan simple supported guideway and investigated its dynamic behavior. Vesnitskii and Metrikine [21] dealt with the parametric resonance of a mass moving on a tensioned string lying on a periodically varying Winkler foundation. In this paper the stability condition was derived theoretically by means of the perturbation approach. Instability analysis for a discretely supported string was made by Metrikine [22]. In general, the damping in the wheel/track interaction system can suppress the instability. Hence, to the authors' knowledge, such a phenomenon has never been reported. Nevertheless, since a railway track has a periodicity due to the discrete sleeper support, the possibility of occurrence of this unstable resonance is inhered in this structure. Therefore, it will be worthwhile from the railway dynamics point of view to investigate this phenomenon. Verichev and Metrikine [23] attempted to assess the instability of an interaction system consisting of a moving mass and an infinite Euler–Bernoulli beam supported by a harmonically varying Winkler foundation. Influences of the mass and damping on the stability were studied based on a theoretical approach as in Ref. [21]. They found that the speed range in which the stability is lost is very narrow, and consequently this effect is insignificant. Since the periodicity in their model is given by a harmonic inhomogeneous stiffness under the assumption of the perturbation analysis, the influence of the parametric feature on the response will be rather moderate. However, in the case of a discretely supported model, the variation in the dynamic equivalent stiffness along a sleeper bay is larger than the harmonically varying case. Therefore the unstable speed range will be widened. While in the above papers