Journal of Sound and Vibration 333 (2014) 3459–3478

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

The efficient computation of the nonlinear dynamic response of a foil–air bearing rotor system

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ARTICLE INFO

Article history:
Received 11 July 2013
Received in revised form
27 February 2014
Accepted 3 March 2014
Handling Editor: W. Lacarbonara
Available online 8 April 2014

ABSTRACT

The foil–air bearing (FAB) enables the emergence of oil-free turbomachinery. However, its potential to introduce undesirable nonlinear effects necessitates a reliable means for calculating the dynamic response. The computational burden has hitherto been alleviated by simplifications that compromised the true nature of the dynamic interaction between the rotor, air film and foil structure, introducing the potential for significant error. The overall novel contribution of this research is the development of efficient algorithms for the simultaneous solution of the state equations. The equations are extracted using two alternative transformations: (i) Finite Difference (FD); and (ii) a novel arbitrary-order Galerkin Reduction (GR) which does not use a grid, considerably reducing the number of state variables. A vectorized formulation facilitates the solution in two alternative ways: (i) in the time domain for arbitrary response via implicit integration using readily available routines; and (ii) in the frequency domain for the direct computation of self-excited periodic response via a novel Harmonic Balance (HB) method. GR and FD are cross-verified by time domain simulations which confirm that GR significantly reduces the computation time. Simulations also cross-verify the time and frequency domain solutions applied to the reference FD model and demonstrate the unique ability of HB to correctly accommodate structural damping.

1. Introduction

A major disadvantage of the conventional self-acting air (or “gas”) bearing is the requirement for a very tight radial clearance for air pressure generation, since shaft growth (e.g. due to temperature) may exceed this clearance [1]. A FAB (or “gas foil” bearing) overcomes this problem by utilising a flexible foil structure to replace the rigid bearing surface (Fig. 1(a)). While stationary, there is either a slight clearance or a preload between shaft (journal) and bearing. As the shaft rotates, the pressure generated pushes the foil boundary away, allowing the shaft to become completely airborne. Advances by *NASA* in the foil materials have opened the way for oil-free high-temperature turbomachines [2], resulting in intensified research into their dynamic performance.

FABs, like gas or oil bearings, are capable of introducing undesirable nonlinear effects into the dynamics of a rotor-bearing system [3]. This necessitates a means for calculating the nonlinear dynamic response of rotor systems with FABs. In the case of a rotordynamic system with incompressible fluid (oil) bearings, the number of state variables is simply $2H$, where H is the total number of rotor modes considered, since the Reynolds Equation (RE) governing the pressure

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http://dx.doi.org/10.1016/j.jsv.2014.03.001
0022-460X/

Nomenclature

$\begin{bmatrix} a_{1,1} & \dots & a_{1,n} \\ \vdots & \vdots & \vdots \\ a_{m,1} & \dots & a_{m,n} \end{bmatrix}(:) \quad = \begin{bmatrix} a_{1,1} & \cdots & a_{m,1} & \cdots & \cdots & \cdots & a_{1,n} & \cdots & a_{m,n} \end{bmatrix}^{\mathrm{T}}$

$[a_1 \cdots a_m]^{\mathrm{T}}.*[b_1 \cdots b_m]^{\mathrm{T}} \quad = [a_1 b_1 \cdots a_m b_m]^{\mathrm{T}}$

$[a_1 \cdots a_m]^{\mathrm{T}} \cdot /[b_1 \cdots b_m]^{\mathrm{T}} \quad = [a_1/b_1 \cdots a_m/b_m]^{\mathrm{T}}$

diag($[a_1 \cdots a_m]^{\mathrm{T}}$) $m \times m$ diagonal matrix with $a_1, \ldots, a_m$ on leading diagonal

cos($[a_1 \cdots a_m]^{\mathrm{T}}$) $[\cos(a_1) \cdots \cos(a_m)]^{\mathrm{T}}$ (same for sin)

$[\ ]_{m\times n}$ matrix [] of size $m \times n$

blk diag($\mathbf{A}, \mathbf{B}, \ldots$) block diagonal matrix containing $\mathbf{A}$,$\mathbf{B}$,…

$(\bar{\ })$ mean term of HB Fourier series of ( ) (Eqs. (38))

$(\ )_{C,q}$, $(\ )_{S,g}$ cos, sin terms of qth harmonic of ( ) (Eqs. (38))

$(\ )'$ differentiation with respect to τ

$\mathbf{A}_\zeta$, $\mathbf{A}_\theta$ constant matrices, Eq. (11b)

$\mathbf{b}_\theta$, $\mathbf{b}_\xi$, $\mathbf{b}_{\xi,\mathrm{ext}}$ vectors defined in Eqs. (20), (35a), and (35b)

$\mathbf{B}^{(1)}_{Q_1,Q_2}$, $\mathbf{B}^{(2)}_{Q_1,Q_2}$ matrices defined in Eqs. (B2a) and (B2b)

c radial clearance (m)

C_n, $A_{n,m}$, $B_{n,m}$ GR coefficients, Eq. (12)

$\mathbf{C}_{Q_1,Q_2}$ constant matrix used in Eq. (42)

$\mathbf{D}$ diagonal matrix, Eq. (24)

$\mathbf{D}_{\psi\theta}$, $\mathbf{D}^{(2)}_{\psi\theta}$, $\mathbf{D}_{\psi\zeta}$, $\mathbf{D}^{(2)}_{\psi\zeta}$ FD matrices used in Eqs. (9)

$\mathbf{E}_h$, $\mathbf{E}^{(2)}_h$ FD matrices used in Eqs. (9)

$\mathbf{f}_g$ vector defined in Eq. (25)

$f_{g,2}$,$f_{g,3}$ second and third elements of $\mathbf{f}_g$

$\mathbf{f}_\psi$, $\mathbf{f}_\mathbf{w}$, $\mathbf{f}_\varepsilon$ vector functions of HB Eqs. (40)

$F_{x,y}$, $F^{\mathrm{FD}}_{x,y}$, $F^{\mathrm{GR}}_{x,y}$ bearing forces and their FD, GR approximations

$\mathbf{g}_{\mathrm{FD}}$, $\mathbf{g}_{\mathrm{GR}}$ right hand of air film state equations in FD and GR

h, $\tilde{h}$, $\tilde{h}_{i,j}$ air film thickness (m), h/c, $\tilde{h}(\zeta_i, \theta_j)$ respectively

$\tilde{\mathbf{h}}_\theta$ $N_\theta \times 1$ film thickness vector, Eq. (8b)

$\tilde{\mathbf{h}}$ $N_z N_\theta \times 1$ film thickness vector (before Eq. (7))

$\tilde{\mathbf{h}}_{\mathrm{coeff}}$ vector defined in Eq. (28)

H number of rotor modes

i,j identifiers for FD grid points

$\mathbf{I}_\theta$,$\mathbf{I}_\xi$ matrices of kernel integrals used in GR, Eqs. (34)

$\mathbf{I}_{P\times P}$ $P \times P$ identity matrix

k_b, $\tilde{k}_b$ foil stiffness per unit area (N/m^3), $k_b c/p_a$ respectively

$\mathbf{K_o}$ constant matrix used in Eq. (33)

$\mathbf{K_r}$, $\mathbf{K_\psi}$ and $\mathbf{K_h}$ constant matrices used in Eq. (36)

L bearing axial length

m_r rotor mass per bearing (kg)

n,m counters for GR expansion in z and θ directions Eqs. (13) and (14)

$\mathbf{n}_{\zeta\theta}$, $\mathbf{n}_\theta$ $N_z N_\theta \times 1$, $N_\theta \times 1$ vectors of ones

N, M order of GR (maxima of n,m)

N_z, N_θ number of points in FD grid in z and θ directions

p, $\tilde{p}$ absolute pressure (Pa), p/p_a respectively

p_a atmospheric pressure (Pa)

$\tilde{p}_g$ non-dimensional gauge pressure ($\tilde{p}-1$)

$\tilde{p}_{g,\theta}$ average $\tilde{p}_g$ in z direction for given θ (Eq. (5))

$\tilde{\mathbf{p}}_g$, $\tilde{\mathbf{p}}_{g,\theta}$ vectors of specific values of $\tilde{p}_g$, $\tilde{p}_{g,\theta}$ (Eqs. (11a) and (11c))

q counter for harmonics in HB expansion (Eqs. (38))

Q order of HB (maximum q)

Q_1,Q_2 two specific values of Q

$\mathbf{r}$ vector defined in Eq. (31b)

R radius of journal (m)