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Influence of potential function asymmetries on the performance of nonlinear energy harvesters under white noise

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ABSTRACT

To improve the broadband transduction capabilities of vibratory energy harvesters (VEHs) under random and non-stationary excitations, many researchers have resorted to purposefully introducing nonlinearities into the restoring force of the harvester. While performing this task, it is often very challenging to maintain a perfectly symmetric restoring force which yields a VEH with an asymmetric potential energy function. This paper investigates the influence of potential function asymmetries on the performance of nonlinear VEHs under white noise inputs. To that end, a quadratic nonlinearity is introduced into the restoring force and its influence on the mean output power of the harvester for mono- and bi-stable quartic potentials is investigated. It is shown that, for VEHs with a mono-stable quartic potential function, the mean output power increases with the degree of potential function asymmetry. On the other hand, for energy harvesters with a bi-stable quartic potential function, asymmetries in the restoring force appear to worsen performance especially for low to moderate noise intensities. When the noise intensity becomes sufficiently large, the influence of the potential function's asymmetry on the mean power diminishes. Results also reveal that a VEH with a symmetric bi-stable quartic potential function produces higher mean power levels than the one with the most asymmetric mono-stable potential. As such, it is concluded that a VEH with a symmetric bi-stable potential is most desirable to improve performance under white noise.

1. Introduction

A significant portion of the current literature on energy harvesting focuses on investigating the design of energy harvesters with a nonlinear restoring force. When carefully introduced, stiffness type nonlinearities can be used to extend the effective steady-state bandwidth of the harvester to a wider range of frequencies which can be used to potentially improve performance in random vibratory environments.

Nonlinearities are usually introduced using external design means leading to two major classes of nonlinear energy harvesters [1–12]. The first class contains all harvesters with a mono-stable potential function and for which the frequency–response curve mimics the typical response of a uni-modal (mono-stable) Duffing oscillator with a hardening/softening

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nonlinearity [2–4]. Examples of mono-stable vibratory energy harvesters (VEHs) include the magnetically levitated inductive harvester proposed by Mann and Sims [3], the axially loaded piezoelectric beam harvester proposed by Masana and Daqaq [4], and the cantilever beam with a tip magnet proposed by Barton et al. [2].

The second class contains all harvesters with a double-well potential energy function and for which the frequency response mimics the response of a bi-modal (bi-stable) Duffing oscillator. Several variances of bi-stable VEHs were proposed [1,2,4,6,7,9] with the main concept of operation remaining the same. The device usually consists of a nonlinear oscillator with two stable equilibria separated by a potential barrier. When the device is subjected to small input excitations, the dynamics remain confined to one potential well exhibiting a nonlinear resonant behavior similar to that of a regular mono-stable Duffing oscillator. However, when enough energy is supplied to allow the dynamic trajectories to overcome the potential barrier and perform inter-well oscillations, the harvester can exhibit complex dynamic responses which can, under some conditions, be favorable for energy harvesting [6,9,13].

Various researchers have analyzed the response behavior of nonlinear VEHs with a *symmetric* potential function to harmonic excitations [3–6,9,10], as well as colored, and white noise [7,14–24]. In most realistic design situations however, maintaining a perfectly *symmetric* potential energy function is a very challenging task. For instance, in buckled-beam type harvesters which incorporate mechanical or magnetic loads, structural imperfections, initial curvature, added masses, as well as asymmetries in the externally applied magnetic fields can produce asymmetry in the potential energy function. When the potential function is asymmetric, the harvester's response can be very different from its symmetric counterpart especially under white noise because the response does not necessarily have a zero mean value. This can significantly influence the harvester's performance and its efficacy.

To better understand the influence of potential function asymmetries on the response of nonlinear energy harvesters under white noise, Halvorsen [25] considered an asymmetric quartic potential and showed that asymmetries can have a significant influence on the output power. In this work, we build on Halvorsen's initial study and investigate the influence of potential asymmetries on the mean power for different values of the quadratic and cubic nonlinearity coefficients, different noise intensities, and for both mono- and bi-stable quartic potentials. In the case of the mono-stable potential, statistical linearization is used to obtain approximate analytical expressions for the response statistics including the mean output power. For the bi-stable potential, finite element methods (FEMs) are used to obtain an approximate numerical solution of the Fokker–Planck–Kolmogorov (FPK) equation governing the response probability density function (PDF). The PDF is then used to study the influence of potential function asymmetry on the mean power.

To achieve the objectives of this work, the rest of the paper is organized as follows. Section 2 presents a general electromechanical model that can be used to study the response of nonlinear mono- and bi-stable energy harvesters. In Section 3, statistical linearization is used to delineate the influence of asymmetries on the mean output power of mono-stable VEHs. Section 4 employs FEMs to solve the FPK equation for the approximate PDF of bi-stable harvesters. The approximate PDF is then used to understand the influence of potential function asymmetries on the mean power. Section 5 presents the important conclusions.

2. Basic model

A physics based model of a nonlinear VEH which consists of a mechanical oscillator coupled to an electric circuit through an electromechanical coupling mechanism is considered. The mechanism can either be piezoelectric, Fig. 1(a), or electromagnetic, Fig. 1(b). The equations of motion can be written in the following general form:

$$m\ddot{\bar{x}} + c\dot{\bar{x}} + \frac{d\bar{U}(\bar{x})}{d\bar{x}} + \theta\bar{y} = -m\ddot{\bar{x}}_b, \tag{1a}$$

$$C_p\dot{\bar{y}} + \frac{\bar{y}}{R} = \theta\dot{\bar{x}} \ \text{(piezoelectric)}, \quad L\dot{\bar{y}} + R\bar{y} = \theta\dot{\bar{x}} \ \text{(electromagnetic)} \tag{1b}$$

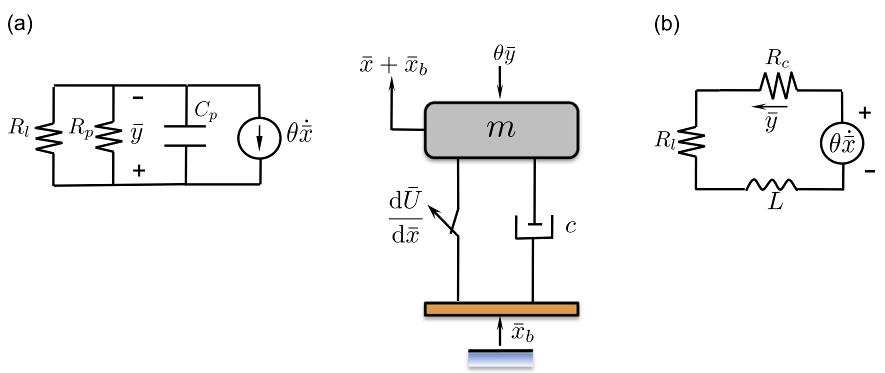

Fig. 1. A simplified representation of a vibratory energy harvester. (a) Piezoelectric and (b) Electromagnetic.