



Numerical investigation of coexisting high and low amplitude responses and safe basin erosion for a coupled linear oscillator and nonlinear absorber system



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ABSTRACT

Over the last half century, numerous nonlinear variants of the tuned mass damper have been developed in order to improve attenuation characteristics. In the present study, the performance of a linear oscillator and an absorber with a strongly nonlinear cubic stiffness is evaluated by using numerical methods. This configuration has been of recent interest due to its capability of wide-band energy absorption. However, high amplitude solutions, which would amplify the response of the system, have been shown to often coexist with the low amplitude solutions. The present research is focused on numerically determining the relative strength of the coexisting solutions. Erosion profiles are presented, quantifying the integrity of the system, i.e. the likelihood of converging to a safe, low amplitude response, and providing an indication of the structural safety of a practical absorber system. The results indicate that the high amplitude solutions not only exist but *significantly influence* the response of the system within the range of expected operating conditions, particularly at excitation frequencies lower than the natural frequency of the linear oscillator. The erosion profiles indicate a 20–40% increase in system integrity for the case of zero damping compared to a small amount of damping, no significant integrity change when adding a small linear stiffness component to the nonlinear absorber, and no significant change in integrity between the midpoint and extreme of the bi-stable range. Additional higher-period solutions are also discovered and evidence of a chaotic response is presented.

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1. Introduction

Nonlinear absorbers have been shown to possess some unique benefits when used as passive vibration absorbers. Under certain conditions, however, nonlinear absorbers are capable of exhibiting multiple stable responses, some of which can be harmful for the primary oscillator. The existence and magnitude of the various solutions in these bi-stable regions have been previously identified, but the basins which indicate the relative strength of each attractor have not yet been studied. In the present paper, the basin erosion profiles of a linear oscillator coupled with an absorber having a nonlinear cubic stiffness are

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constructed. In this section, a review of related literature is first presented. Then, key methodology related to the construction and interpretation of basins and their corresponding erosion profiles is discussed.

1.1. Literature review

A recurring challenge in the design of structures and mechanical systems is the effective attenuation of vibrations resulting from internal imbalance or external excitation. When it is not possible to remove the source of excitation or adjust system parameter values to be less susceptible to vibration, an auxiliary absorber may be beneficial. One of the most common auxiliary absorbers is the *tuned mass damper* (TMD), patented by Frahm [1]. Analytical relations describing the motion of the TMD were published soon after by Ormondroyd and Den Hartog, and are still widely used today [2,3]. The performance benefits of the TMD have been demonstrated for random excitation of single *degree-of-freedom* (DOF) systems [4] and multiple-DOF systems [5]. Many parametric studies have been conducted, for example optimizing the damping ratio for narrow-band attenuation [6] and determining favorable system parameters for wide-band [7] and random excitation [8]. The behavior and capabilities of TMDs are now well understood and as a result these devices are now widely implemented in a variety of structural and mechanical systems.

Over the years, many attempts have been made to improve upon the performance of the TMD by introducing various nonlinear elements. Of particular interest to the present work are those absorbers which incorporate a nonlinear stiffness element, hereafter simply referred to as *nonlinear TMDs* (NTMDs). An NTMD with a cubic stiffness force profile was first proposed by Roberson as a way to increase the effective bandwidth over that of a TMD [9]. Pipes later derived analytical solutions for an NTMD with a hyperbolic sine force profile which reduced the gradient of the resonant peaks in the frequency domain [10]. However, arguably the most significant potential of the NTMD was not realized until more recently when the “energy pumping” phenomenon was discovered.

Energy pumping refers to the rapid and irreversible transfer of energy between two oscillators, the theory of which is rooted in Nayfeh and others' work regarding energy transfer between nonlinear normal modes [11,12]. Gendelman utilized the energy pumping phenomenon to develop a nonlinear absorber capable of irreversibly extracting energy from a primary structure [13]. Along with his colleagues, Gendelman expanded on this initial work by presenting a thorough explanation of the dynamics governing the energy transfer later the same year [14–16].

Although the NTMD has been demonstrated to exhibit unique benefits as a vibration absorber, it is now well known that in some cases dangerous high amplitude solutions exist in addition to the low amplitude “safe” solutions [17,18]. In a related paper by the authors, the results of a numerical study for the use of a series STMD to attenuate the response of a linear oscillator and a TMD which has been detuned due to a weak hardening type nonlinearity were presented [19]. An alternate method of eliminating the high amplitude solutions by using a piecewise-quadratic viscous damper was investigated by Starosvetsky and Gendelman [20].

In contrast to the previous literature which has identified the *existence* of various coexisting solutions in the NTMD system, the focus of the present paper is to identify the basins of attraction which indicate the *strength* of each solution. Basin portraits are constructed and various integrity measures are used to construct the *erosion profiles*, which concisely illustrate the degradation of the safe basin versus excitation magnitude.

1.2. Background

The thorough study of the erosion profiles in the present work is computationally expensive, and results can only be efficiently obtained and concisely presented by utilizing some of the key concepts presented in this section. The parallelized multi-degrees-of-freedom cell mapping method, a technique for efficiently calculating the basins of attraction of higher-dimensional systems, is first discussed. Then, four scalar integrity measures used to condense the extensive data into a small number of erosion profiles are introduced.

1.2.1. Parallelized multi-DOF cell mapping

A well-known characteristic of nonlinear dynamic systems is that they are capable of possessing multiple coexisting solutions, thus creating *bi-stable regions*. For this reason nonlinear systems are often avoided for practical applications. However, if certain performance benefits can outweigh the burden of a complex response, it is important to characterize any existing bi-stable regions in order to most accurately predict the system behavior.

Characterizing the behavior within the bi-stable regions consists of two steps: identifying the amplitude of each stable attractor and determining the size and distribution of each existing *basin*, the set of initial conditions $\mathbf{q}_0$ converging to each attractor. The former can be accomplished by using numerical continuation techniques or approximate analytical methods. In the present case, AUTO bifurcation and continuation software, which employs a pseudo-arclength continuation method, is used [21]. The latter is accomplished by constructing *basin portraits*, two dimensional cross-sections of the n -dimensional basins of attraction. The basin portrait planes are denoted by Σ^{ab} , the superscripts identifying the two dimensions a and b in which the planes lie.

The most straightforward method used to construct basin portraits is the *grid of starts* (GOS) [22]. In this method, a grid of *initial conditions* (IC) within the portrait plane is defined and from each initial point a solution is obtained by using numerical integration. The steady-state behavior of each solution identifies to which attractor the initial point converges

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