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On the asymptotic solution of sound penetration into a rigid porous half-plane: A modified saddle point method

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ABSTRACT

An approximate analytical formula has been derived for the prediction of sound penetration into a semi-infinite rigid porous medium due to a monopole source. The sound fields can be expressed in an integral form that is amenable to analytical and numerical analyses. A modified saddle point method is applied to evaluate the integral asymptotically, which leads to a closed-form expression. The validity of the asymptotic formula is confirmed by comparison with the numerical results computed by the fast field formulation and the direct evaluation of the integral. The present analytical formula has been demonstrated to be sufficiently accurate to predict the penetration of sound into the semi-infinite rigid porous medium.

1. Introduction

In recent years, porous materials have increasingly been used in conjunction with a complex structure in support of sound absorption and sound insulation for passive noise control in enclosures. They have found applications in the automotive and aerospace industries, buildings, and many types of industrial machinery. Many numerical tools have been developed to assist the design of porous materials coupled with the strategic use of multilayer structures and cavities [1–4]. In recent years, there is a trend of developing hybrid sound absorbers (smart absorbing materials) that combines a porous layer of sound absorbent materials with a piezoelectric actuator acting as an active control noise source [5–7]. Often, the acoustical characteristics of the porous materials are needed to enable successful modeling of poroelastic media in a complex structure and a cavity backing.

The information on the acoustical characteristics of sound absorption materials has different levels of complexity depending on the models used in the analysis [1]. It varies from the most sophisticated poroelastic model based on the Biot theory [8,9] to the simplest locally reacting impedance model. A commonly used model assumes the modified fluid approach where the frame of the porous material is motionless, i.e., a rigid porous medium. This model as well as the Biot model has an advantage in that the visco-inertial and thermal dissipations can be modeled in the system. The acoustic and non-acoustic (microscopic) properties of the porous materials are needed; these may be determined by an inversion method where the properties are adjusted in the model to match the acoustic measurement results. Most of these inverse methods [10,11] are based on impedance tube measurements where only the macroscopic acoustic parameters at normal incidence can be determined.

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It is well known that the microscopic properties of the porous materials are dependent only on the geometry in the local scale of the micro-structural parameters [12]. It will be useful to deduce these macroscopic parameters from the measurement of sound fields at the near-grazing propagation both above and within the rigid materials [13]. An accurate computation of sound fields above and below the rigid porous material is therefore needed. To this end, an accurate and efficient Green's function is frequently needed for use in many boundary element methods for calculating the sound field scattered by buried objects [14].

Li and Liu [15] obtained an asymptotic solution for the sound penetration into the rigid porous ground by a double saddle point method. They used the pole subtraction method to improve the accuracy of the asymptotic solution when the pole lies close to the saddle point. However, their solution has an irremovable singularity when the source is directly above (or below) the observer. In this paper, a modified saddle point method [16–18] is exploited, which gives higher accuracy of the sound fields even when the pole is located in close proximity to the saddle point. The asymptotic solution according to the modified saddle point method will be compared with the accurate wave-based numerical solutions.

2. Theoretical formulation

Suppose an airborne source is placed above a porous half-plane where the upper medium is air with density ρ and sound speed c. The lower medium is a porous half-space with complex density ρ_1 and sound speed c_1. The interface between the air and the porous half-plane is located at the $z=0$ plane. Fig. 1a shows a schematic diagram of the problem. It is useful to define the respective ratio of densities and sound speeds as

$$\zeta = \rho/\rho_1 \tag{1a}$$

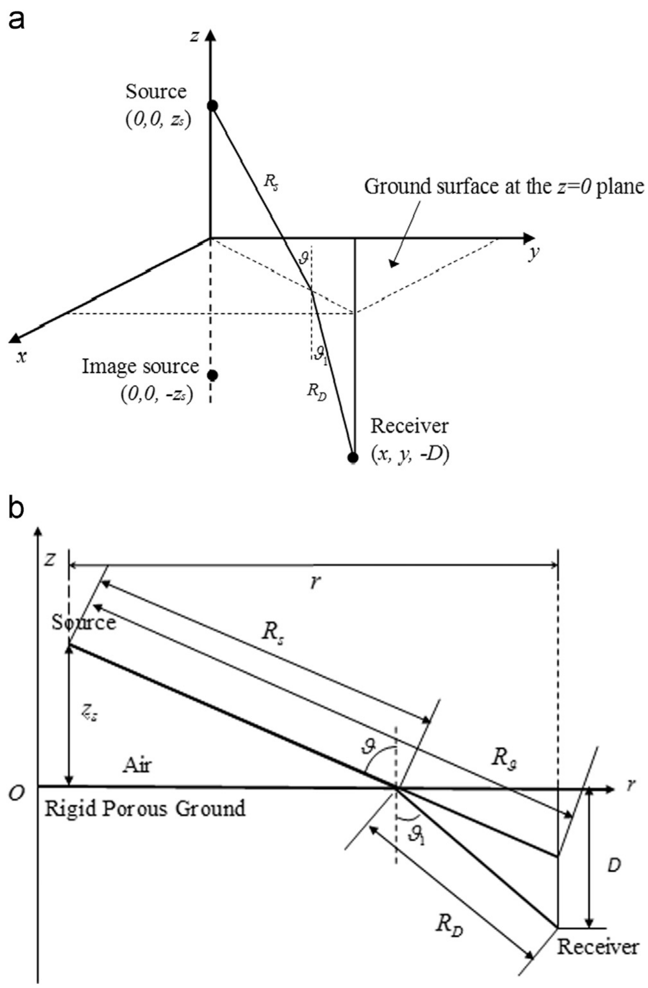

Fig. 1. (a) A schematic diagram of the problem when the source is located above the porous medium, but the receiver is located below the rigid porous medium. (b) an illustration of different geometrical parameters used in the theoretical formulation.