



Contents lists available at SciVerse ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

A new nonlinear higher-order shear deformation theory with thickness variation for large-amplitude vibrations of laminated doubly curved shells

M. Amabili*

Canada Research Chair (Tier 1) Department of Mechanical Engineering, McGill University, Macdonald Engineering Building, 817 Sherbrooke Street West, Montreal, Quebec, Canada H3A 0C3

ARTICLE INFO

Article history:

Received 7 November 2012

Received in revised form

21 February 2013

Accepted 27 March 2013

Handling Editor: M.P. Cartmell

ABSTRACT

A consistent higher-order shear deformation nonlinear theory is developed for shells of generic shape allowing for thickness variation by using six variables; geometric imperfections are also taken into account. The geometrically nonlinear strain–displacement relationships are derived retaining full nonlinear terms in the in-plane displacements. They are presented in curvilinear coordinates in a formulation that can be readily implemented in computer codes. This new theory is applied to laminated circular cylindrical shells complete around the circumference and simply supported at the ends. Linear (natural frequencies) and geometrically nonlinear (large-amplitude forced response) vibrations are studied by using the present theory and results are compared to those obtained by using the refined Amabili–Reddy higher-order shear deformation nonlinear shell theory, which neglects thickness variations.

© 2013 Elsevier Ltd. All rights reserved.

1. Introduction

Thickness variation can be important for shells in case of (i) high-frequency vibrations and (ii) for soft materials, e.g. biological tissues. In fact, (i) modes with dominant thickness oscillation can appear in the high-frequency range and (ii) biological tissues are often subjected to very large strains associated with very large thickness variation.

Classical shell theories, which neglect shear deformation and rotary inertia, give inaccurate natural frequencies for moderately thick or laminated anisotropic shells and plates (e.g. [1]). In order to overcome this limitation, shear deformation theories have been introduced, but they still neglect thickness variation. These theories can be classified as first-order and higher-order shear deformation theories [2]; in the first category, a shear correction factor is required for the equilibrium since a uniform shear strain is assumed through the shell thickness. Higher-order shear deformation theories overcome this limitation since a realistic shear stress distribution through the shell thickness is assumed, which also satisfies the condition of zero shear stresses at both top and bottom shell surfaces.

Several higher-order shear deformation shell theories have been proposed. Librescu [3] developed a nonlinear shell theory by expanding the shell displacements with cubic terms in the transverse coordinate. A linear higher-order shear deformation theory of shells has been introduced by Reddy [4] and Reddy and Liu [5]. Arciniega and Reddy [6] have improved the theory developed in [5]. A review of shell theories has been presented by Reddy and Arciniega [7].

*Tel.: +1 514 398 3068; fax: +1 514 398 7365.

E-mail address: marco.amabili@mcgill.ca

Reddy [8] developed the nonlinear higher-order shear deformation theory of plates, taking into account von Kármán type nonlinear terms. Dennis and Palazotto [9] and Palazotto and Dennis [10] have extended Reddy shell theory [4] to nonlinear deformation by introducing the von Kármán type nonlinear terms. These theories have been also discussed in the books of Amabili [2] and Reddy [11]. In the existing higher-order shear deformation geometrically nonlinear shell theories, the von Kármán type nonlinear terms (i.e. those involving the normal displacement only) have been added to the linear equations, so that consistent derivation has not been performed. Moreover, the von Kármán type nonlinear terms are known for being accurate for classical shell theories only for small displacements and moderately small rotations. Therefore, it is important to derive a consistent higher-order shear deformation theory that keeps all the nonlinear terms in the normal and in-plane displacements. For this reason, a new nonlinear higher-order shear deformation theory that retains in-plane nonlinear terms has been recently derived by Amabili and Reddy [12] by using a consistent approach. This theory belongs to the class of the equivalent single layer (ESL) theories and has the novelty to retain nonlinear terms with in-plane displacements, neglected in other formulations. It must be observed that another class of theories, the layer wise model (LWM) has also been developed, but not retaining geometrically nonlinear terms, see e.g. Carrera [13].

Amabili [14] and Alijani and Amabili [15] have applied the theory developed in Ref. [12] to laminated closed and open circular cylindrical shells and have shown that it gives an important accuracy improvement for thick laminated deep shells, in particular for vibration modes with a low number of circumferential waves, with respect to commonly used nonlinear higher-order shear deformation theories. The theory developed by Amabili and Reddy [12] introduces the artificial constraint of no thickness deformation, reducing the problem to five variables (3 displacements and two rotations). However, they discussed that their new theory could be extended to take into account thickness variation.

An accurate linear shell theory that takes into account thickness variation has been developed by Carrera et al. [16,17]. In particular, in Ref. [17] natural frequencies of laminated spherical and cylindrical panels are investigated by using this theory. The effect of transverse normal stress in linear vibrations of laminated shells and plates has been investigated by Carrera [13] by using a model taking into account continuity of interlaminar transverse and shear stresses and zigzag form of the displacement distribution in the shell thickness.

Nayak et al. [18] and introduced a new shell finite element containing three translations, two rotations of the normals about the shell mid-surface, and one drilling rotational degree of freedom per node. This 4-node element has been used to study the transient dynamic response of composite shells. Awrejcewicz et al. [19] studied nonlinear vibrations of shells of complex shape and variable thickness. Many other studies on vibrations and nonlinear vibrations of laminated shells and plates are available in the literature, e.g. [20,21].

In the present study, a consistent higher-order shear deformation nonlinear theory is developed for shells of generic shape allowing for thickness variation by using six variables; geometric imperfections are also taken into account. The geometrically nonlinear strain–displacement relationships are derived retaining full nonlinear terms in the in-plane displacements. They are presented in curvilinear coordinates in a formulation that can be readily implemented in computer codes. This new theory is applied to laminated circular cylindrical shells complete around the circumference and simply supported at the ends. Linear (natural frequencies) and geometrically nonlinear (large-amplitude forced response) vibrations are studied by using the present theory and results are compared to those obtained by using the refined Amabili–Reddy higher-order shear deformation nonlinear shell theory [12], which neglects thickness variations.

2. Nonlinear higher-order shear deformation theory with thickness variation

A laminated shell of arbitrary shape, made of a finite number of orthotropic layers, oriented arbitrarily with respect to the shell principal curvilinear coordinates (α_1, α_2) , is considered, as shown in Fig. 1. The development of the theory remains the same for shells made of isotropic, orthotropic or functionally graded materials. The displacements of an arbitrary point (α_1, α_2) on the middle surface of the shell are denoted by u, v and w , in the α_1, α_2 and z directions, respectively; w is taken positive outward from the center of the smallest radius of curvature. Initial geometric imperfections of the shell associated with zero initial tension are denoted by displacement w_0 in normal direction, also taken positive outward. The thickness h of the shell is assumed to be small compared to the principal radii of curvature of the shell, so that only moderately thick shells can be considered. The displacements (u_1, u_2, u_3) of a generic point (see Figs. 1 and 2) are related to the middle surface displacements by

$$u_1 = (1 + z/R_1)u + z\phi_1 + z^2\psi_1 + z^3\gamma_1 + z^4\theta_1, \quad (1a)$$

$$u_2 = (1 + z/R_2)v + z\phi_2 + z^2\psi_2 + z^3\gamma_2 + z^4\theta_2, \quad (1b)$$

$$u_3 = w + z\chi + w_0, \quad (1c)$$

where ϕ_1 and ϕ_2 are the rotations of the transverse normals at $z=0$ about the α_2 and α_1 axes, respectively, and χ is the thickness variation per unit thickness. Then $\psi_1, \psi_2, \gamma_1, \gamma_2, \theta_1$ and θ_2 are functions to be determined in terms of u, v, w, ϕ_1 and ϕ_2 . Thus, the six variables describing the shell deformation are u, v, w, ϕ_1, ϕ_2 and χ . In Eqs. (1) the in-plane displacements have been expanded up to the 4th order in z while the normal displacement has been assumed to be linear in z . Obviously, a more refined expression of the variation of the normal displacement can be introduced, see e.g. Carrera [13], including zig-zag functions, which can be piece-wise linear functions with slope discontinuity at the interlaminar interface, and higher

Download English Version:

<https://daneshyari.com/en/article/10289225>

Download Persian Version:

<https://daneshyari.com/article/10289225>

[Daneshyari.com](https://daneshyari.com)