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Frequency response design of uncertain systems using performance indices and meta-models

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ABSTRACT

Systems that are operated near their resonance frequencies experience vibrations that can lead to impaired performance, overstressing, fatigue fracture and adverse human reactions. Frequency response (FR) analysis can be invoked to mitigate the effects. When components of a system are described by random variables, modal frequencies and modal shapes, or, amplitudes and phases, are also random variables and the frequency response (FR) design of the system becomes more complex since it requires the solution of a frequency-variant probability problem. This paper presents a methodology to provide the frequency response design of uncertain systems using a transfer function approach. The methodology is found to be robust, expandable and flexible and can be applied to multi-disciplinary systems with n-dof and multiple design constraints. The novelty of the approach is the creation of a frequency-invariant probability problem through: (a) the discretization of the frequency band of interest into multiple contiguous point frequencies, (b) the introduction of new performance indices that measure the probability of success over the entire frequency band, and (c) the introduction of explicit meta-models to provide sufficiently fast probability evaluations through Monte Carlo simulation. The key to the performance indices are limit-state functions formed at all discrete, contiguous, frequencies. Each limit-state function establishes a conformance region in terms of the random design variables. The probabilities of the conformance regions are correctly combined to provide a single series-system index to be maximized by adjusting distribution parameters. The simple explicit meta-model is based on Kriging and performance measures at arbitrary design sets are efficiently calculated. Error analysis suggests ways to predict and control the errors with regards to meta-model fitting and probability calculations and so the method appears sufficiently accurate for engineering applications. The proposed methodology has applications in numerous areas such as electrical filters and structural mechanics – all with n-dof and multiple responses. The Performance indices can be evaluated at any frequency over any number of frequency ranges. A case study of a vibration absorber mechanism shows how the new methodology provides an improved and timely design with controllable accuracy when compared with previous proposals that employed modal frequencies.

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1. Introduction

Design of vibratory systems involves finding the values of key components to correctly position the amplitudes and phases to best meet upper or lower specifications over a range of frequencies. Usually, the design variables can be

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partitioned into (a) noise variables wherein the uncertainty is always present and (b) the control variables that can be adjusted through upfront design and include for example, sizes, weights, and material compositions, or more often, the lumping of these characteristics into a component parameter.

For real physical systems, experimentation to improve the design is costly and time consuming, especially when the experimental space is large. In response to this, mathematical models, or mechanistic models have been developed and computer simulations run to provide the behaviour. For dynamic systems the model typically comprises a set of differential equations in either the state-space form or the equations of motion form Refs. [1,2]. For example, the equation of motion for a linear mechanical system with excitation is

$$\mathbf{M}(\mathbf{v})\ddot{\mathbf{r}}+\mathbf{B}(\mathbf{v})\dot{\mathbf{r}}+\mathbf{K}(\mathbf{v})\mathbf{r}=\mathbf{P}_0(t) \tag{1}$$

where $\mathbf{r}$ represents displacement and $\mathbf{v}$ the design variables that comprise mass, stiffness and damping. For harmonic excitations the function of time is a sinusoid. One way to find responses is through Modal analysis that leads to the following standard eigenvalue equations

$$[\mathbf{K}(\mathbf{v})-\omega_l^2\mathbf{M}(\mathbf{v})]\boldsymbol{\psi}_l=0 \tag{2}$$

where ω_l is the lth natural frequency of the system and $\boldsymbol{\psi}_l$ is the corresponding mode shape. The response can be written as functions of ω_l and $\boldsymbol{\psi}_l$ [3,4]. It is clear that for uncertainty in the system the natural frequencies (and mode shapes) are also uncertain and approaches in Refs. [5,6] address this issue. Alternatively, from Eq. (1) and through network analysis [7] or dynamic systems theory [2–4], we can develop a set of implicit equations in the frequency domain of the form

$$\mathbf{F}(\mathbf{v},j\omega)\mathbf{z}(j\omega)=\mathbf{P}_0 \tag{3}$$

where $\mathbf{z}$ denotes the response vector and $\mathbf{P}_0$ the amplitude vector. The solution to Eq. (3) can always be found numerically. However, for a reasonable sized system, a transfer function for a selected response denoted as $G(\mathbf{v},j\omega)$ gives

$$\begin{aligned}|G(\mathbf{v},\omega)|&=\frac{a}{P_0}\\ \angle G(\mathbf{v},\omega)&=\phi\end{aligned} \tag{4}$$

where a is the amplitude and ϕ is the phase. The selected response is completely described by $|G(\mathbf{v},\omega)|$ and $\angle G(\mathbf{v},\omega)$. Now, to obtain the complete frequency response ω is varied and tables or graphs of a/P_0 and ϕ are the end result. In the probabilistic situation, the uncertain design variables, now, denoted as $\mathbf{V}$, imply that both $|G(\mathbf{V},\omega)|$ and $\angle G(\mathbf{V},\omega)$ are uncertain over frequency [8,9]. That is, given the joint probability distributions of the design variables $f_{\mathbf{V}}(\mathbf{v})$, there will be probability density functions along the frequency spectrum ($\boldsymbol{\omega}$). For example, the amplitude relation gives the frequency-variant density function p as

$$|G(\mathbf{V},\boldsymbol{\omega})|=p(f_{\mathbf{V}}(\mathbf{v}),\boldsymbol{\omega}) \tag{5}$$

Although Monte Carlo simulation (MCS) always works to evaluate probabilities [6], when the mechanistic model is complex and simulations are time consuming, the design process becomes onerous and thus a faster, meta-model approach has been investigated [10–20]. The idea is to rationally select a few so-called training sets that represent the variability of the design variables, then simulate the outcomes and finally fit the data with a simpler, explicit, model. Research has compared the performance of various meta-models in fitting complex functions and discussed their advantages and disadvantages. The most common meta-models available are the regression models, radial basis functions and the Kriging models [10–20].

The concept and application of performance indices are well developed in control system design in the time-domain [21]. In general the indices measure the difference between the actual response and an ideal case over the cycle time. Examples include, integral of squared error (ISE) and integral of time squared error (ITSE) and so-forth. Performance indices for uncertain transient problems are given in Ref. [22]. In the frequency domain, the concepts of band-pass and band-reject play a central role in electrical filter design. Therein an ideal performance specification – usually in terms of amplitude – is invoked and the design ensures that the performance measure meets the specification. For uncertain systems Fricker et al. [9] introduces the concept of an envelope to capture variations at a set of extreme frequencies. In this paper, we advance the concept of performance indices and invoke meta-models to develop frequency-based performance indices that provide a novel frequency response design method for systems with uncertainty. The significance of the proposed methodology is that it sets the foundation for estimating the uncertainty while still in the design stage. A methodology is presented in the form of flow chart, as shown in Fig. 1, to address these issues.

For ease of presentation of the above steps, consider the spring-mass-damper system in Fig. 2. The amplitude of the mass is the performance measure and its transfer function is

$$G(\mathbf{v},j\omega)=\frac{1}{-\omega^2 m+j\omega c+k} \tag{6}$$

Then following Eq. (4) the normalized amplitude (i.e. divide by P_0/k) is

$$a(m,c,k,\omega)=k\left(\frac{1}{(k-m\omega^2)^2+c^2\omega^2}\right)^{1/2} \tag{7}$$

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