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Stochastic joint time–frequency response analysis of nonlinear structural systems

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ABSTRACT

A novel approximate analytical approach for determining the response evolutionary power spectrum (EPS) of nonlinear/hysteretic structural systems subject to stochastic excitation is developed. Specifically, relying on the theory of locally stationary processes and utilizing a recently proposed representation of non-stationary stochastic processes via wavelets, a versatile formula for determining the nonlinear system response EPS is derived; this is done in conjunction with a stochastic averaging treatment of the problem and by resorting to the orthogonality properties of harmonic wavelets. Further, the nonlinear system non-stationary response amplitude probability density function (PDF), which is required as input for the developed approach, is determined either by utilizing a numerical path integral scheme, or by employing a time-dependent Rayleigh PDF approximation technique. A significant advantage of the approach relates to the fact that it is readily applicable for treating not only separable but non-separable in time and frequency EPS as well. The hardening Duffing and the versatile Preisach (hysteretic) oscillators are considered in the numerical examples section. Comparisons with pertinent Monte Carlo simulations demonstrate the reliability of the approach.

1. Introduction

Monte Carlo simulation (MCS) based approaches are among the most versatile tools for general uncertainty quantification of systems of engineering interest. Nevertheless, there are cases where the computational cost can be prohibitive, especially when large scale complex systems are considered. Thus, there is a need for developing efficient approximate numerical and/or analytical methodologies for addressing problems set in a stochastic framework (e.g. see [1–3] for some recent work).

In this regard, a sustained challenge in the field of nonlinear stochastic dynamics has been the determination of the response power spectrum of nonlinear systems subject to stochastic excitation. Further, it is noted that the determination of the nonlinear system response power spectrum is also of great value to experimental dynamics since it is strongly related to quantities commonly measured in structural dynamic tests (e.g. [4]). In fact, experimental results suggest that certain systems exhibiting significant nonlinear behavior possess response power spectra which demonstrate considerable broadening and shift in the resonance peaks in comparison with the response power spectra of the corresponding linear systems (e.g. [5,6]).

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Clearly, stochastic approaches capable of predicting accurately the aforementioned phenomena are required. Indicatively, research work towards determining the response power spectrum of nonlinear engineering systems includes a limited number of exact solutions (e.g. [7,8]), the development of a quasi-linearization approach which employs the Ito differential rule (e.g. [9]), moment closure schemes (e.g. [10,11]), approaches based on the Fokker–Planck (F–P) equation (e.g. [12,13]) and methodologies which rely on the Volterra series theory (e.g. [14–20]).

Further, Miles [21,22] developed an improved linearization approach and derived an approximate formula for determining the nonlinear system response power spectrum. The approach utilizes the concept of conditional power spectrum and succeeds in predicting the correct peak values and bandwidths, while circumventing limitations of the conventional statistical linearization approach (e.g. [23,24]). Interpreting the approach, it considers a family of equivalent linear systems whose elements are response amplitude envelope dependent. The nonlinear response power spectrum is determined as a weighted sum of the response power spectra of the linear systems, whereas the probability density function (PDF) of the response amplitude process serves as the weight function. Recently, Spanos et al. [25] provided with an alternative perspective on the veracity of the approach and derived a concrete proof of the formula.

Furthermore, the performance of the approach was enhanced to capture higher harmonics via a stochastic averaging treatment (e.g. [26,27]), whereas the methodology was extended to address oscillators with a nonlinear asymmetrical restoring force [28], continuous systems [29] and multi-degree-of-freedom (MDOF) systems [30]. Also, the approach has been applied for determining the response power spectrum of complex hysteretic (i.e. Preisach formalism) systems (e.g. [31]). Moreover, Krenk and Roberts [32] and Rudinger and Krenk [33,34] developed an alternative formulation based on local similarity between the random response and the deterministic response at the same energy level of the corresponding undamped oscillator. It was further extended to address systems with parametric excitations [35]. Overall, the approach has demonstrated considerable accuracy and versatility and has been utilized successfully in various engineering applications such as in the context of system identification (e.g. [36–39] and of earthquake engineering [40].

Nevertheless, the existing research work and the validity of the described spectral approach are restricted to stationary stochastic processes only. In this regard, structural systems are often subject to extreme events and severe excitations such as seismic motions, winds, hurricanes, ocean waves, tsunamis, blasts and impact loads which inherently possess the attribute of evolution in time. Thus, representation of these phenomena by non-stationary stochastic processes is necessary to capture accurately the system/structure behavior. Although several research efforts have focused on determining the non-stationary response of nonlinear systems under stochastic excitation, limited results exist in the context of a stochastic joint time–frequency response analysis (e.g. [41]).

In this paper, a novel approximate analytical approach for determining the nonlinear system response evolutionary power spectrum (EPS) is developed based on the theoretical framework of locally stationary processes and on the orthogonality properties of harmonic wavelets. It can be viewed as an extension and generalization of the aforementioned spectral approach (e.g. [25]) to account for non-stationary processes of arbitrary EPS. It is noted that the non-stationary response amplitude PDF naturally takes the place of the weight function in this generalized version of the formula. In this regard, efficient techniques, such as the numerical path integral, are employed to determine the response amplitude PDF. Numerical examples include nonlinear systems comprising the versatile Preisach formalism, recently applied in modeling the hysteretic behavior of smart materials. The reliability of the approach is demonstrated by comparisons with pertinent Monte Carlo simulation data.

2. Mathematical formulation

2.1. Locally stationary wavelet process and harmonic wavelets

In this section a brief overview of the basic properties of harmonic wavelets is provided and essential elements of the locally stationary wavelet (LSW) process representation are reviewed.

In this regard, the family of generalized harmonic wavelets (e.g. [42]) utilizes two parameters (m, n) for the definition of the bandwidth at each scale. The main advantage of this family relates to the decoupling of the time–frequency resolution achieved at each scale from the value of the central frequency; this is not the case with other commonly used wavelet bases such as the Morlet and other families.

Further, generalized harmonic wavelets have a box-shaped frequency spectrum, whereas a wavelet of (m, n) scale and (k) position in time attains a representation in the frequency domain of the form

$$\Psi^{G}_{(m,n),k}(\omega) = \begin{cases} \frac{1}{(n-m)\Delta\omega}\exp\left(-i\omega\frac{kT_0}{n-m}\right), & m\Delta\omega \le \omega \le n\Delta\omega \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where m, n and k are considered to be positive integers and

$$\Delta\omega = \frac{2\pi}{T_0}, \tag{2}$$

where T_0 is the total time duration of the signal under consideration. A collection of harmonic wavelets of the form of Eq. (1) spanning adjacent non-overlapping intervals at different scales along the frequency axis is shown schematically in Fig. 1.

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