Journal of Sound and Vibration 332 (2013) 5462–5475

Contents lists available at SciVerse ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Torus-doubling bifurcations and strange nonchaotic attractors in a vibro-impact system

Yongxiang Zhang [a,*], Guanwei Luo [b]

[a] College of Science, Shenyang Agricultural University, Shenyang 110866, China
[b] School of Mechatronic Engineering, Lanzhou Jiaotong University, Lanzhou 730070, China

ARTICLE INFO

Article history:
Received 18 February 2013
Received in revised form
22 April 2013
Accepted 29 April 2013
Handling Editor: L.N. Virgin
Available online 4 June 2013

ABSTRACT

Whether strange nonchaotic attractors (SNAs) can occur typically in dynamical systems other than quasiperiodically driven systems has been an open problem. Here we show that the SNAs can be induced by the interruption of torus-doubling bifurcation in a periodically driven vibro-impact system. It is found that the creation of SNAs occurs due to the collision of doubled torus with some unstable periodic orbits. Based on the Poincaré map technique, two types of codimension-3 bifurcations are represented and typical torus-doubling bifurcations can occur in the neighborhood of these bifurcation points. We use the singular continuous Fourier spectrum and its scaling to verify the strangeness (nondifferentiability) of the attractors. The implicitly transcendental map is used to calculate the largest Lyapunov exponent by QR-based algorithm, which ensures that the underlying dynamics (with nonpositive Lyapunov exponents) is nonchaotic. The associated mechanism is described for the creation of SNAs through detecting unstable periodic orbits from Poincaré map series.

© 2013 Elsevier Ltd. All rights reserved.

1. Introduction

SNAs are considered as typical structures of quasiperiodically forced nonlinear systems. The phrase strange nonchaotic attractors refers to an attractor that is nonchaotic in the sense that its orbits are not exponentially sensitive to perturbation (i.e., its largest Lyapunov exponent is either zero or negative), but the attractor is strange (nondifferentiability) in the sense that its phase space structure has nontrivial fractal properties. Since the pioneering work of Grebogi et al. in 1984 [1], the subject of SNAs has attracted continuous interest in the nonlinear and statistical physics community [2–5, references therein]. Further, SNAs have a practical application in secure communication [6–8]. For SNAs, it is noted that SNAs can be typical in the quasiperiodic systems. A subject of intense further interest is the way in which the truncation of period doubling creates SNAs. The number of torus-doubling bifurcations in the sequence depends upon the amplitude of the external quasiperiodic force. For the case of sufficiently large amplitudes, a simple smooth torus may transform into an SNA. For small amplitude values, several torus-doubling bifurcations may occur before the SNA arises. Thus, an important issue is to understand the reason for the termination of the torus-doubling cascades in the quasiperiodically forced systems. It has been found that the creation of SNAs often occurs due to the collision of a period-doubled torus with its unstable parent torus [9], or gradual fractalization of the torus [10–12]. The fractalization route for the formation of SNAs can be investigated by identifying unstable sets in SNAs, and it is argued that the quasiperiodic analog of crisis causes fractalization of the torus

* Corresponding author. Tel.: +86 02488493201.
E-mail addresses: zyx9701@126.com, zyx9701@syau.edu.cn (Y. Zhang).

0022-460X/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.jsv.2013.04.046

to create SNAs [13]. The torus-doubling sequence is tamed due to a subharmonic bifurcation leading to the creation of SNAs [14]. Apart from the creation of SNA due to the collapse of the tori, it is also shown that there are some regions of the system parameters where the torus-doubling bifurcation is truncated by a merging bifurcation leading to the formation of a torus bubble in low dimensional systems [15]. In a quasiperiodically forced map, the interruption of torus doubling bifurcations and the genesis of SNAs have been analyzed [16]. Most studies of SNAs have been on low-dimensional quasiperiodically driven systems. One interesting question relates to the need for external forcing. In the known examples the external drive is quasiperiodic in time: it is not clear whether this form of drive is necessary for the creation of strange nonchaotic motion or merely sufficient. Can SNAs be expected to occur more commonly in dynamical systems than previously thought, e.g., nonsmooth systems or hybrid dynamical systems [17,18]?

The aim of this paper is to answer the question: are SNAs restricted to quasiperiodically driven nonlinear systems or low-dimensional systems? We verify that the SNAs can be observed in a vibro-impact system with periodically driven forcing. We consider the periodically three-degree-of-freedom vibro-impact system which is a representative model for the nonsmooth systems or the hybrid dynamical systems. Another interesting question relating to the origins of such dynamics (SNAs) in such a nonsmooth system remains open. For chaotic attractors, there can in principle be several positive Lyapunov exponents (the phenomenon of hyperchaos). Although the SNAs are geometrically similar to chaotic attractors, any further analogy between them is clearly not possible, and thus it is of interest to investigate the creation of SNAs and the nature of SNAs in high-dimensional dynamical systems. Impacts give rise to nonlinearity and discontinuity so that the vibro-impact system can exhibit rich and complicated dynamical behaviors. Research into the dynamical behaviors of vibro-impact systems has important significance on optimization design of machinery and noise suppression. Early studies on vibro-impact dynamics including the singularities [19–22], global bifurcations [23,24] and routes to chaos [25] have been investigated. In recent years, various bifurcations of vibro-impact systems have received great attention, such as period-doubling bifurcation [26], Hopf bifurcation [27,28] and several types of codimension-2 bifurcations [29–31]. In general, each of these bifurcations has important observable physical consequence. More complicated and interesting dynamics are likely to occur in the neighborhood of the bifurcation points [32,33].

In this paper, we consider a three-degree-of-freedom vibro-impact system [33] and we first focused on torus-doubling bifurcations in the neighborhood of codimension-3 bifurcation points. Local codimension-3 bifurcation of this system called the Hopf–Hopf–Flip bifurcation, concerning two complex conjugate pairs of eigenvalues and a negative eigenvalue of linearized map escaping the unit circle simultaneously, are investigated by using the Poincaré maps and the numerical integral method. Another type of codimension-3 bifurcation is referred to as the Hopf–Hopf bifurcation in there order strong resonant case. In the usual case, the interruption of torus doubling bifurcations cannot occur in the vibro-impact systems [26]. In high-dimensional phase space, it is difficult to present a collision of a period-doubled torus with its unstable parent torus. Next, we are interested in the interruption of torus doubling bifurcation and genesis of SNAs in such a nonsmooth system. We found that the birth of SNAs is due to the collision of doubled torus with some unstable periodic orbits. As regard as the vibro-impact system, there is no known technique for determining the strangeness of attractors. Because the vibro-impact systems are nonsmooth, the derivative $\partial x/\partial\theta$ does not exist and the partial sums computed by (11) [34] cannot be obtained by an analytical expression. Therefore, the phase sensitivity exponent method [34] is invalid. For the two-dimensional maps, the analytical results is presented indicating that the capacity (or box-counting) dimension of the SNAs is two while their information dimension is one [35]. Hunt and Ott present a rigorous result supporting the idea for a simple class of quasiperiodically forced dynamical systems [11]. The rigorous results are also presented demonstrating that the SNAs' box-counting dimension is larger than their information dimension by 1 [12]. However, for the high dimensional vibro-impact systems, it is not easy to get the analytical results due to the non-smoothness. In order to verify the existence of SNAs in the vibro-impact system, singular continuous Fourier spectra and its scaling are used to verify the strangeness (nondifferentiability) of the attractors [36–38]. We develop the singular continuous Fourier spectra method based on the Poincaré map technique. To ensure the underlying dynamics is nonchaotic, the calculation of Lyapunov exponents is necessary. Though the calculation method is not straightforward under the consideration of non-smooth elements, the good news is that there are some studies on calculation of Lyapunov exponents in the vibro-impact systems [39,40]. In our model, the implicitly transcendental map is used to calculate the largest Lyapunov exponent by QR-based algorithm, which ensures that the underlying dynamics is nonchaotic (the largest Lyapunov exponent is nonpositive). In order to understand the underlying mechanism of the termination of the torus-doubling cascades in the vibro-impact system, we have detected some unstable periodic orbits embedded in SNAs. Such unstable periodic orbits are similar to the unstable periodic orbits of chaotic dynamical systems [41,42]. We can detect some unstable periodic orbits of the Poincaré map in the vibro-impact system by solving the unconstrained optimization problems of scalar function [41] when the SNAs occur for some fixed parameter values. This type of SNAs can thus be expected to occur more commonly in other nonsmooth systems.

2. The mechanical model, Poincaré map, singular continuous spectra and unstable sets

The mechanical model for a three-degree-of-freedom vibro-impact system is shown in Fig. 1. It is a representative model for the hybrid dynamical systems or the nonsmooth systems. This type of system is often encountered in practice, for instance in the models of hammer-like devices. The model is widely used to illustrate the rich dynamics, e.g., codimension-2 bifurcations and phase-locking dynamics [29,33]. The masses M_1 and M_2 are connected to linear springs with stiffness K_1 and K_2, and linear viscous dashpots with damping constants C_1 and C_2. The excitation on mass M_1 is harmonic with

Download English Version:

https://daneshyari.com/en/article/10289296

Download Persian Version:

https://daneshyari.com/article/10289296

Daneshyari.com