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A high-order harmonic balance method for systems with distinct states

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ABSTRACT

A pure frequency domain method for the computation of periodic solutions of nonlinear ordinary differential equations (ODEs) is proposed in this study. The method is particularly suitable for the analysis of systems that feature distinct states, i.e. where the ODEs involve piecewise defined functions. An event-driven scheme is used which is based on the direct calculation of the state transition time instants between these states. An analytical formulation of the governing nonlinear algebraic system of equations is developed for the case of piecewise polynomial systems. Moreover, it is shown that derivatives of the solution of up to second order can be calculated analytically, making the method especially attractive for design studies.

The methodology is applied to several structural dynamical systems with conservative and dissipative nonlinearities in externally excited and autonomous configurations. Great performance and robustness of the proposed procedure was ascertained.

1. Introduction

In the fields of science and engineering, a common task is the calculation of periodic solutions of nonlinear ordinary differential equations. In our study, we will focus on ODEs of arbitrary dimension involving generic, i.e. possibly strong and non-smooth nonlinear functions. In particular, we will address systems that can comprise distinct states so that the nonlinear functions are only piecewise defined. In mechanical engineering, such nonlinearities arise e.g. in structural systems with contact joints, where stick, slip and lift-off are often considered as locally distinct states [1]. In electrical engineering, examples for such systems are electrical circuits, where e.g. transistors, rectifiers and switches induce distinct system states. A rheological example is superelastic shape memory alloys where the phases and phase transformations between e.g. martensite and austenite phases can be regarded as distinct states [2]. Of course, many other examples can be found in various fields of science and engineering.

In order to find periodic solutions to such problems, analytical approaches are often not applicable and computational methods have to be employed. Besides the family of time integration methods, so-called frequency domain methods are commonly used due to their often superior computational efficiency. The basic idea of frequency domain methods is to choose a truncated Fourier ansatz for the dynamic variables, thereby exploiting the periodic nature of the solution. This class of methods gives rise to nonlinear algebraic systems of equations. Depending on whether the solution is sought in the frequency domain or in a collocated time domain, and whether the residual is formulated in the frequency or time domain, the methods can be grouped into (multi-)harmonic balance method [3,4], trigonometric collocation method [5] and time

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spectral method [6]. Among these methods, the multi- or high-order harmonic balance method (HBM) is probably the most commonly applied method.

For the HBM, it is generally necessary to compute the spectrum of the nonlinear function that governs the ODE. This task can generally be performed by different methods. In the following, we will focus on those methods that are capable of treating systems with distinct states.

The Alternating-Frequency-Time (AFT) scheme [7] is one of the most commonly applied approaches in this context. The AFT scheme involves a sampling of the nonlinear function and subsequent back-transformation into frequency domain. Advantages of this method are the broad applicability, the comparatively small implementation effort and the low computational effort for evaluating the residual function. The latter aspect is particularly true if the (Inverse) Fast Fourier Transform is used for the transformation between time and frequency domain. A drawback is that nonlinearities with distinct states involve special treatment. A sampling of the nonlinear function is not straight-forward, because the current state at a specific time instant is not always a priori known. So-called predictor–corrector schemes [8] are frequently employed to perform the switching between different states for these systems. In classical AFT schemes, the sampling points are fixed, and do not need to coincide with the state transition time instants. This inherently induces discretization errors. Hence, the sensitivity of the transition time instants with respect to arbitrary parameters cannot be captured accurately, resulting in inaccurate derivatives, in particular for higher-order derivatives.

More recently, a purely frequency-based formulation was proposed by Cochelin and Vergez [9]. The authors applied the Asymptotic Numerical Method to expand the periodic solution into a power series based on high-order derivatives of the nonlinear function. In order to obtain these derivatives efficiently, a so-called quadratic recast is performed where the original system of equations is transformed into a system of only quadratic order. An advantage of this method is the computationally robust and efficient continuation of the solution. A drawback is obviously the required quadratic recast which can be difficult for generic types of nonlinear functions. Moreover, systems with distinct states need to be artificially smoothed in order to accomplish a closed-form quadratic recast. This smoothing procedure induces inaccuracies compared to the original non-smooth model.

In order to avoid the shortcomings of a required recast or the degenerated accuracy due to sampling, a pure frequency domain formulation for the original system with distinct states can instead be used. Such an approach necessarily involves the direct calculation of the transition time instants between the states. For high-order HBM, these approaches have only been developed for special types of nonlinearities so far. For example Petrov and Ewins [10] developed an analytical formulation of the HBM for piecewise linear friction interface elements in structural dynamical problems. In this study, the approach in [10] is extended to generic systems with an arbitrary number of distinct states, see Section 2. Analytical formulations can be developed in case of piecewise polynomial systems, as it will be shown in Section 2.5. Moreover, the formulation facilitates the analytical calculation of gradients of up to second order as an inexpensive postprocessing step, see Section 2.4. To demonstrate the capabilities and the performance of the proposed methodology, several numerical examples are studied in Section 3. Finally, conclusions are drawn in Section 4.

2. Methods of analysis

2.1. Harmonic balance method for systems with distinct states

Consider a system whose dynamics can be described by a first-order ordinary differential equation,

$$\dot{\mathbf{y}} = \mathbf{f}(\mathbf{y}, t), \tag{1}$$

in which $\dot{()}$ denotes derivative with respect to time t. It is assumed that the generally nonlinear function $\mathbf{f}$ is piecewise defined within closed regions of the state space of $\mathbf{y}$. These closed regions in state space are denoted *states* throughout this paper. These states shall not be confused with the vector $\mathbf{y}$ which is sometimes also referred to as state in literature since it represents a point in state space.

As time evolves, the system can assume several states, see Fig. 1. A transition between these states is termed state transition and the corresponding time instant is called state transition time instant in the following. The system enters a specific state at the corresponding transition time t^- and leaves it at t^+. Each possible state k consists of a nonlinear function $\mathbf{f}_k$,

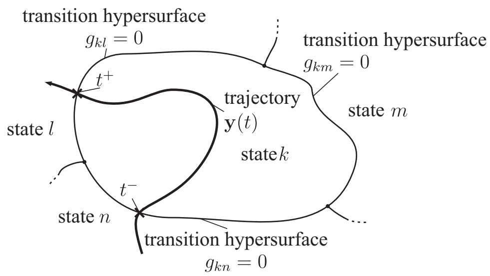

Fig. 1. Illustration of the dynamics of a system with distinct states.