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Fundamental mode shape and its derivatives in structural damage localization

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ABSTRACT

Various techniques for structural damage detection involving modal parameters have widely been used over the past few decades. This is because the modal parameters of a structure can easily be obtained from forced, free or ambient vibration measurements. In many of these techniques, mode shape curvature has been used for localization of damage. In this paper, a mathematical basis is provided to show the correlation between a structural damage and a change in the fundamental mode shape and its derivatives. This has been achieved by deriving the expression of a damaged mode shape utilizing a perturbation approach. For a cantilever shear beam, discretized into a large number of elements, this approach demonstrates that the change in the fundamental mode shape due to any damage is an excellent indicator of damage localization as it is found to be discontinuous at the location of damage. Further, the change in higher derivatives (i.e., slope and curvature) of the fundamental mode shape is shown to be sensitive enough in damage localization. A numerical study involving a shear building and a steel moment-resisting frame is conducted to show the effectiveness of the proposed approach in damage localization. It has been found from this study that spline fitting of mode shapes in case of a limited number of degrees-of-freedom, which is generally adopted for plotting mode shapes and their derivatives, may lead to false detection of damage.

1. Introduction

Vibration-based damage detection techniques are gaining popularity among researchers due to their non-destructive nature and potential to achieve a continuous structural health monitoring at a low cost. Structural damage may be defined as any change in the properties of a system that affects the system's performance. The severity and nature of these damages can vary considerably. In some cases, the damage may be minor and of least significance, while in other cases the damage may influence significantly the performance of the system. An early detection of a damage in terms of location and severity is always desirable in order to prohibit catastrophic failure of a structure.

Mode shapes are generally used in structural damage detection and localization rather than natural frequencies as frequency-based methods are found to be less sensitive towards structural damage. In the last couple of decades, various mode shape-based approaches have been proposed for damage detection and localization. Pandey et al. [1] proposed mode shape curvature to be a sensitive parameter for damage localization. Mode shape curvature was calculated in this study using the central difference approximation and utilized for damage localization of a simulated beam discretized into a number of finite elements. Sampaio et al. [2] extended the idea of Pandey et al. [1] by applying the curvature-based method

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to frequency response function instead of mode shape and demonstrated the potential of this approach by considering real data. Wahab and Roeck [3] used mode shape curvature for health inspection of a real prestressed concrete bridge structure. Later, Wahab [4] utilized mode shape curvature along with natural frequencies and mode shapes for finite element model updating. According to Abdo and Hori [5], modes with more deviation in frequency will have larger variation in position vectors and hence, will be more sensitive to damage.

Kim et al. [6] conducted a study to identify the sensitivity of frequency and mode shape for damage detection using a finite element model of a prestressed concrete beam. Wu and Law [7] extended the mode shape curvature-based concept into two-dimension to localize damage in a plate structure by using uniform load surface (ULS) curvature obtained by Chebyshev polynomial approximation. Sazonov and Klinkhachorn [8] investigated the optimal sampling locations for damage detection in order to maximize spatial resolution and to provide maximum sensitivity to damage in presence of measurement noise.

Qiao et al. [9] used the mode shape curvature-based techniques to detect delamination in composite plate by developing ULS from simulation as well as experimental measurements. Lee and Eun [10] used mode shapes and their derivatives obtained from experimental results of static and dynamic tests to study the behavior of a structure under damaged condition. Whalen [11] used higher derivatives of mode shapes on a simple beam with various boundary conditions to identify damage. Catbas et al. [12] illustrated the robustness and strength of flexibility-based curvatures using experimental data from a steel grid structure. Ray-Chaudhuri [13] investigated the behavior of eigenproperties, namely frequencies and mode shapes, in presence of stiffness degradation in simulated structures such as shear building and steel moment-resisting frame (SMRF). A mathematical formulation with the help of perturbation approach was developed in this study to inspect the change in frequencies as well as mode shapes for a given location of stiffness degradation. It was observed in this study that the mode with zero crossing near the location of damage is more sensitive towards damage. It was also established that the higher modes are more sensitive to damage as they have more zero crossings. Higher modes are however difficult to obtain for a typical civil engineering structure as these modes are unlikely to get excited under ambient conditions due to higher input energy requirements. Thus, the fundamental mode is more frequently used for damage detection [14].

Cao and Qiao [15] proposed a Laplacian scheme along with three different parameters for estimating multi-resolution modal curvatures from the mode shapes to have a better clarity in damage localization. Tomaszewska [16] applied a flexibility-based approach and a mode shape curvature-based approach on the results from a simulated simply supported beam and a real historical tower building to study the effect of statistical error. Zhu et al. [14] demonstrated the efficiency of the change in slope of the fundamental mode shape as a damage indicating feature by performing a numerical study on a eight-story shear building and conducting experiments on a three-story building model. Further, the magnitude of the damages was quantified using this mode shape slope-based feature. An iterative scheme was developed in this study to localized the damage. Radzienski et al. [17] performed an experiment on an aluminium cantilever beam to identify the modal parameters. It was found in this study that mode shape curvature is an important parameter for damage localization. Dilena et al. [18] demonstrated that mode shape curvature can be a useful term for damage location on a reinforced concrete single span bridge. Bai et al. [19] applied mode shape curvature-based technique on a two-dimensional plate grid structure to locate the damage in the structure. Recently, Xiang et al. [20] decomposed the curvature of mode shapes using wavelet transform to localize crack in a conical structure. Lu et al. [21] identified multiple cracks in beams with the help of a response sensitivity analysis followed by a mode shape curvature-based technique.

It is now clear from the aforementioned literature that the mode shape-based methods are gaining popularity among researchers for damage detection and localization. Although such methods have widely been applied on numerical models, experimental data or real structures for damage detection and localization, as per the authors' knowledge, no closed form expression has yet been presented to establish a correlation between the change in mode shape or its derivatives with the location of damage.

In this study, using first-order perturbation approach, a closed form solution is derived for change in mode shape due to a localized structural damage. Considering the fundamental mode, the effect of damage is then interpreted with the change in mode shape, slope of mode shape, and curvature of mode shape. A numerical illustration is then presented to show the robustness of this change in mode shape and its derivatives for damage localization. For this purpose, numerical models of a shear building and a steel moment-resisting frame (SMRF), both 12-story high, have been considered, and damage was introduced in the form of a stiffness degradation in an intermediate story.

2. Formulation of damaged mode shape

Let us consider the case of free vibration of a multi-degree-of-freedom (MDOF) shear building with an initial (undamaged) stiffness matrix of $[\mathbf{k}]$. The equation of motion relating the lumped mass matrix $[\mathbf{m}]$ and the damaged (reduced) stiffness $[\hat{\mathbf{k}}]$ can then be expressed in the following form:

$$[\mathbf{m}]\{\ddot{y}(t)\} + [\hat{\mathbf{k}}]\{y(t)\} = 0 \tag{1}$$

where $[\hat{\mathbf{k}}] = [\mathbf{k}] + [\Delta\mathbf{k}]$ with $[\Delta\mathbf{k}]$ being the change in stiffness due to damage. Now for small $[\Delta\mathbf{k}]$, using the perturbation approach as given in Ray-Chaudhuri [13], the ith damaged orthonormal (mass-normalized) mode shape $\hat{\phi}^{(i)}$ can be related to