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Autoparametric resonances in a structure/fluid interaction system carrying a cylindrical liquid tank

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Abstract

The nonlinear-coupled vibrations of an elastic structure and liquid sloshing in a cylindrical container are investigated. The behavior of the liquid surface is governed by a kind of the Mathieu equation because the structure is subjected to a vertical and sinusoidal excitation. Modal equations for liquid sloshing governing the coupled motions are derived when the natural frequency of the structure is equal to twice the natural frequency of an anti-symmetric mode of sloshing. The theoretical resonance curves are determined by using van der Pol's method. The influences of a liquid level and a detuning parameter on the theoretical resonance curves are investigated when only the excitation frequency is selected as a control parameter. The inclination of a frequency response curve depends on the liquid level. Furthermore, a small deviation of the tuning condition may cause amplitude- and phase-modulated motions and chaotic vibrations. This deviation also leads to separate the occurrence region of the coupled vibration into two regions of the excitation frequency. The theoretical resonance curves are quantitatively in agreement with the experimental data. Lastly, the amplitude- and phase-modulated motions and chaotic vibrations were observed in experiments.

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Nomenclature

c	damping coefficient of the structure
F_l	vertical fluid force
F_0	amplitude of excitation
g	acceleration of gravity
h	depth of the liquid
k	spring constant of the structure
M	summation of masses of the structure and the liquid ($= m + m_l$)
m	mass of the structure
m_l	mass of the liquid
P	fluid pressure
p_0	natural frequency of the structure (or the main system)
p_{mn}	natural frequency of the (m, n) sloshing mode (m, n: integers)
R	tank radius
(r, θ, z)	circular cylindrical coordinate system (see Fig. 1)
t	time
(x, y, z)	rectangular coordinate system (see Fig. 1)
z_0	displacement of the structure
η	displacement of the liquid surface
$v_1 = m/M$	
$v_2 = m_l R/(\pi M h)$	
ρ	fluid density
ϕ	velocity potential
ω	excitation frequency
ζ_{mn}	damping ratios of (m, n) sloshing mode

1. Introduction

Vibrations of an elastic structure with liquid containers are very important problems from the viewpoint of structure performance and disaster prevention. Associated problems often appear in industry such as elevated water tanks and liquefied natural gas (LNG) tanks. When the motion of a liquid surface—referred to as sloshing—occurs in containers, the nonlinearity of the liquid inertia force essentially appears. Therefore, this nonlinearity must be taken into account to accurately analyze the dynamic behavior of the liquid sloshing. Liquid sloshing dynamics including free and forced free-surface motions, sloshing interaction with elastic structures, numerical techniques, and sloshing under low gravitational field are reviewed elsewhere [1].

Many papers examined the nonlinear behavior of liquid sloshing in circular cylindrical containers that are excited harmonically. However, most of these deal with the case of a horizontal excitation [2–5], while few papers have examined a vertical excitation [6–8]. Interaction problems of an elastic structure and a liquid sloshing—which are the main subjects of this paper—have primarily focused on tuned liquid dampers. Regarding the damping effects that circular cylindrical liquid tanks have on the vibration of the structure under horizontal excitation, Senda and Nakagawa [9] were early to report on linear analysis. Since then, experimental [10,11] and nonlinear analyses [12,13] have also been reported. On the other hand, few papers have dealt with interaction problems where a structure is subjected to vertical excitation for a rectangular tank [14] and a circular cylindrical tank [15–18]. The authors of this paper examined the case where an axisymmetric sloshing mode was excited in a circular cylindrical tank [18] and improving the accuracy of the former analysis [15] by considering multiple sloshing modes. However, as two different modes of an identical natural frequency are degenerated, the behavior of an anti-symmetrical sloshing mode in a cylindrical tank becomes more complicated than that of the case of an axisymmetric mode.