Renewable Energy 96 (2016) 137–147

Contents lists available at ScienceDirect

Renewable Energy

journal homepage: www.elsevier.com/locate/renene

Fluid and borehole wall temperature profiles in vertical geothermal boreholes with multiple U-tubes

Massimo Cimmino

Department of Mechanical Engineering, McGill University, 817 Sherbrooke St. W., Montreal, Quebec, H3A 0C3, Canada

ARTICLE INFO

Article history:
Received 23 December 2015
Received in revised form
16 April 2016
Accepted 22 April 2016

Keywords:
Geothermal boreholes
Ground heat exchangers
Borehole thermal resistances
Thermal response factors

ABSTRACT

An analytical model for the calculation of fluid temperature profiles in geothermal boreholes with multiple U-tubes is presented. A linear system of first order differential equations is built based on a steady-state thermal resistances approach to the heat transfer between the U-tube pipes and the borehole wall. Analytical expressions for fluid temperature profiles are provided for boreholes with arbitrary borehole wall temperature profiles as well as piece-wise uniform and uniform borehole wall temperature for different U-tube configurations: independent, parallel and series. The analytical model is coupled to a finite line source model of the heat transfer between the borehole and the ground. Fluid and borehole wall temperature profiles are solved simultaneously using numerical Laplace transforms to consider the time variation of the temperatures and heat extraction rates. Results are verified against a finite difference model of the borehole. Differences between the fluid temperatures calculated with the numerical and analytical models are smaller than 0.003 °C.

1. Introduction

Ground source heat pump (GSHP) systems coupled to vertical geothermal boreholes offer an energy efficient mean of providing heating and cooling in buildings. The energy performance of GSHP systems is directly dependent on the outlet fluid temperature from the bore field and entering the heat pump. The accurate prediction of operating fluid temperatures in geothermal bore fields is therefore essential to the proper design of such systems.

A common strategy for the simulation of vertical geothermal boreholes is to use separate models for the heat transfer inside and outside the borehole, using the borehole wall as an interface between the two models. Cimmino [1] has shown that the fluid and borehole wall temperature variations along the borehole length can have a significant impact on the long-term temperature variations in geothermal bore fields. The analysis was however limited to single U-tube boreholes due to the lack of analytical solutions to the heat interaction between the fluid and the borehole wall for multiple U-tube boreholes and non-uniform borehole wall temperature.

The energy performance of solar assisted GSHP systems with independent double U-tube boreholes has been assessed in the context of residential systems [2–4]. These systems use independent U-tubes for the heat pump and solar collectors, which serve to counter the load imbalance in heating dominated climates and decrease the size of the geothermal system. Existing studies rely on the assumption of uniform borehole wall temperature to calculate the fluid temperature profiles, which might impact on the long-term fluid and ground temperature predictions.

The objective of this paper is to provide an analytical method to calculate fluid temperature profiles and outlet fluid temperatures in geothermal boreholes. The general differential equation for steady-state heat transfer between the fluid flowing through any amount of U-tube pipes and the borehole wall is presented and solved using the matrix exponential. Boundary conditions are applied at the top and bottom of the pipes to consider the cases of independent U-tubes as well as parallel and series configurations. Expressions for the fluid temperature profiles are given for an arbitrary borehole wall temperature profile and for piecewise uniform and uniform borehole wall temperatures. Results are first presented for a uniform borehole wall temperature and verified using a finite difference model for up to 4 pairs of U-tube pipes. The analytical model is coupled to a finite line source model [1,5] to consider the time variation of the borehole wall temperature during heat extraction. The time variation of fluid and borehole wall temperatures are calculated based on given total heat extraction rates in the borehole.

E-mail address: massimo.cimmino@polymtl.ca.

http://dx.doi.org/10.1016/j.renene.2016.04.067

Nomenclature

Variables

α_s	soil thermal diffusivity
Δt	simulation time step
σ	damping coefficient for numerical Laplace transform
$c_{f,i}$	fluid specific heat
D	borehole buried depth
D_s	pipe shank spacing
H	borehole length
k_g	grout thermal conductivity
k_s	ground thermal conductivity
$\dot{m}_i$	fluid flow rate
n_p	number of U-tubes
n_q	number of borehole segments
N_t	number of simulation time steps
r_b	borehole radius
$r_{p,o}$	pipe outer radius
R_p	fluid to outer pipe wall thermal resistance
t	time
$T_b(z)$	borehole temperature profile
T_g	undisturbed ground temperature
(x_i, y_j)	coordinates of pipe axis

Matrices and vectors

$\boldsymbol{\Delta h}(t) = [\Delta h_{uv}(t)]$	matrix of segment-to-segment response factors increments
$\boldsymbol{A} = [A_{ij}]$	coefficient matrix of the system of differential equations
$\boldsymbol{B_{in}}, \boldsymbol{B_{out}}$	coefficient matrices for pipe configurations
$\boldsymbol{D} = \mathrm{diag}(\lambda_i)$	diagonal matrix of the eigenvalues of $\boldsymbol{A}$
$\boldsymbol{E}(z) = [E_{ij}(z)]$	matrix exponential of $\boldsymbol{A}z$
$\boldsymbol{E_{in}}, \boldsymbol{E_{out}}, \boldsymbol{E_b}$	coefficient matrices for boundary condition at $z = H$
$\boldsymbol{\Delta E_{in}}, \boldsymbol{\Delta E_{out}}, \boldsymbol{\Delta E_b}$	coefficient matrices for heat balances over borehole segments
$\boldsymbol{M} = [M_i]$	line vector of $\dot{m}_i c_{f,i}$ in each pipe
$\boldsymbol{q}(z) = [q_j(z)]$	column vector of heat transfer rates per unit length of pipe
$\boldsymbol{Q_b} = [Q_{b,v}]$	column vector of average heat extraction rate per unit length of borehole segment
$\boldsymbol{R} = [R_{ij}]$	matrix of borehole thermal resistances
$\boldsymbol{R}^{-1} = [S_{ij}]$	inverse matrix of borehole thermal resistances
$\boldsymbol{T_b} = [T_{b,v}]$	column vector of average borehole segment temperatures
$\boldsymbol{T_{f,in}}, \boldsymbol{T_{f,out}}$	column vectors of inlet and outlet fluid temperatures
$\boldsymbol{T_f}(z) = [T_{f,j}(z)]$	column vector of fluid temperature profiles
$\boldsymbol{V} = [\boldsymbol{v_j}]$	matrix of eigenvectors of $\boldsymbol{A}$

Indices

i, j	pipe indices
k	time index
u, v	borehole segment indices

1.1. *Literature review*

Solutions for the fluid temperature profiles in single U-tube geothermal boreholes were presented by Eskilson and Claesson [6]. The borehole was modeled as a delta-circuit of thermal resistances between the 2 pipes and the borehole wall. A linear system of 2 first-order differential equations was obtained from thermal energy balances on the fluid in a borehole cross-section. The differential equations were solved using Laplace transforms and expressions for the fluid temperature profiles were given for an arbitrary borehole wall temperature profile. Hellström [7] later presented solutions for the simplified cases of uniform heat transfer rate and uniform borehole wall temperature. Other solutions for single U-tube boreholes using a thermal resistance model for the borehole cross section were presented by Yang et al. [8] and Beier [9].

Zeng et al. [10] used the same solution technique to obtain solutions for the fluid temperature profiles for double U-tube boreholes with symmetrically positioned pipes in parallel and series configurations and uniform borehole wall temperature. Eslami-Nejad and Bernier [2,11] and, more recently, Belzile et al. [3] expanded on the solution to cover double U-tube boreholes with independent circuits and different fluid flow rates.

Al-Khoury [12,13] proposed an analytical model that includes the thermal capacity of the fluid and the grout material. Differential equations for the heat transfer in the fluid, the grout material and the ground are presented and solved using the Discrete Fourier Transform. The analytical model was verified against a numerical finite element model. Al-Khoury and Focaccia [14] applied the same methodology to double U-tube boreholes in parallel configuration and validated the model against results from a thermal response test.

Beier [15] modeled the U-tube pipes in a single U-tube borehole as an equivalent radius pipe positioned at the borehole center. The equivalent pipe is separated at the center to obtain two channels, one for each pipe. The model accounts for the thermal capacity of the fluid and the grout material and assumes radial-only heat transfer in the grout material and the ground. The problem is solved using Laplace transforms and the solution is inverted using the Stehfest [16] Laplace transform inversion algorithm. The model is validated against sandbox data [17] and measurements from thermal response tests [18,19].

Ma et al. [20] used the composite-medium infinite line source solution [21] to model the transient heat transfer process from the U-tube pipes to the ground through the grout material. The pipe temperature in single and double U-tube boreholes is assumed to be uniform and equal for all pipes. The fluid temperature profiles are then calculated from the heat transfer between the fluid and the uniform temperature pipes.

Eslami-Nejad et al. [22,23] calculated the fluid temperature, vapor quality and pressure drop profiles in direct expansion geothermal boreholes using CO$_2$ as the heat carrier fluid. The borehole cross-section was modeled as a circuit of thermal resistances and solved using a finite volume method.

So-called thermal resistance and capacity models (TRCM), or capacity resistance models (CARM), have been proposed by several authors [24–28]. Such models are obtained by adding thermal capacity nodes to the circuit of thermal resistances to model the thermal capacity of the fluid and the grout material and the travel time of the fluid through the U-tubes.

Marcotte and Pasquier [29] used a 3D finite element model to evaluate the fluid temperature profiles in geothermal boreholes. The authors compared the simulated temperature profiles to several assumptions of the fluid temperature variation in the boreholes. It was shown that the arithmetic mean of the inlet and