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Improved blade element momentum theory for wind turbine aerodynamic computations

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ABSTRACT

Blade element momentum (BEM) theory is widely used in aerodynamic performance predictions and design applications for wind turbines. However, the classic BEM method is not quite accurate which often tends to under-predict the aerodynamic forces near root and over-predict its performance near tip. The reliability of the aerodynamic calculations and design optimizations is greatly reduced due to this problem. To improve the momentum theory, in this paper the influence of pressure drop due to wake rotation and the effect of radial velocity at the rotor disc in the momentum theory are considered. Thus the axial induction factor in far downstream is not simply twice of the induction factor at disc. To calculate the performance of wind turbine rotors, the improved momentum theory is considered together with both Glauert's tip correction and Shen's tip correction. Numerical tests have been performed for the MEXICO rotor. Results show that the improved BEM theory gives a better prediction than the classic BEM method, especially in the blade tip region, when comparing to the MEXICO measurements.

1. Introduction

The Blade Element momentum (BEM) theory [1–3] is a widely used and fast method in aerodynamic and aero-elastic applications of wind turbines. The actuator disc (AD) momentum concept and blade element theory are two indispensable parts of the BEM Theory. Rankine firstly introduced the methodology of using actuator disc to represent a rotor in 1865. However, it is Froude who found a feasible physical explanation about actuator disc in 1889 [2]. He showed that the decrease of velocity at the disc equals to the half of that in far downstream. The concept of dividing rotor blade into separate elements was proposed by Drzewiecki in 1892 [4] where he drew the velocity triangle for each element without including velocity induction. With the development of vortex theory, the concept of optimum wind turbine was established. The amount of energy that can be extracted from the wind has a theoretical upper limit, equal to 59%. According to van Kuik et al. [5], it is called Betz–Joukowsky limit. After some further improvements made by Prandtl and Glauert [6], the classic BEM theory was founded.

However, some assumptions in the BEM equations lead to discrepancies between experimental data and BEM results. One of the assumptions is ignoring the static pressure drop caused by wake rotation such that the pressure in far upstream of the rotor equals that in far downstream. Based on this assumption, it is derived that the axial induction at the rotor disc is the half of that in the far wake. Joukowsky [7] considered the effect of wake rotation in the analysis of propellers, which leads to the increase in rotor power coefficient at low tip speed ratio. Wilson and Lissaman [8] adopted wake rotation in the analysis of wind turbine. The axial induction factor at the disc is always smaller than the half of that in infinitely far downstream at tip speed ratio below 2. Vries [9] made a similar analytical study on the consequences of non-rotating wake but he did not apply this to the BEM theory and disregarded the additional pressure change caused by wake rotation. Sharp [10] reiterated the full analysis of the general momentum theory. He came to the conclusion that the rotor power coefficient increases at low tip speed ratio and can exceed the Betz–Joukowsky limit. The equations forming the general momentum theory of wind turbine rotors were clearly analyzed in Sørensen et al.'s paper [11] where they derived a most general momentum theory which can be used to get the famous momentum theories such as the Glauert Model, Burton Model and Joukowsky Model.

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However, the assumption of ignoring radial velocity in the general moment theory also needs to be improved according to the facts. According to the MEXICO experiment [12], a significant radial flow can be seen at places just before the rotor blade. The influences of the radial flow have not been considered in both classic momentum theory and general momentum theory. Only a few researches on this topic can be found in literature. Micallef et al. [13] provided some fundamental insight into the radial velocity very close to the rotor blades, which was not yet explored in literature. They showed that the radial velocity reaches appreciable magnitudes especially in the tip region through experimental and numerical analysis. Madsen et al. [14] made a comprehensive comparison between BEM computations and numerical results from actuator disc Navier-Stokes simulations. They considered the influence of wake rotation and radial flow to the axial velocity. However, the relationship between radial velocity and axial velocity correction is obtained through fitting the results from their AD simulations. The radial velocity was also discussed in the studies [15,25] with the topic of rotational augmentation which mainly occurs on the suction side of a rotating blade section near root. At high angles of attack, increased loading which is bigger than its 2D aerodynamic data is observed in the inboard parts of the blades. The effect of rotational augmentation can be taken into account through correcting the 2D airfoil data and thus is not a focus in the present momentum theory.

The paper is organized as follows. In Section 2, an improved actuator disc momentum theory that includes the influence of wake rotation and radial velocity is derived, which is an extension of the classic momentum theory. The improved momentum theory combined with the blade element theory and tip loss correction is presented in Section 3. Results are presented in Section 4 which compares the improved BEM method with the classic BEM method and the MEXICO experimental data. Finally, conclusions are drawn in Section 5.

2. Improved momentum theory

The BEM theory, consisting of actuator disc momentum theory and blade element theory, is widely used in aerodynamic design and power prediction of a wind turbine rotor. The AD momentum theory represents the momentum balance on a rotating annular stream tube and the blade element theory calculates the forces produced by an airfoil cross-section of a rotating blade. The general momentum theory, which included the pressure difference across the rotor disc due to wake rotation, was considered in the references [5–10]. Some important equations and derivations, which are useful in the derivation of the new AD momentum theory, are summarized here.

2.1. Pressure difference across the actuator disc

The AD model is a simplified model of representing a wind turbine rotor with a circular disc. Extending from the disc upstream and downstream along the streamlines, a stream tube with circular cross-section is created. A control volume is formed by the cylindrical surface of this stream tube and two cross-sections at both ends, which is depicted in Fig. 1. Four axial locations are considered in the momentum analysis of this stream tube: free stream region or far upstream, just before the rotor, just after the rotor and far wake region or far downstream. The wind flows into the control volume with an axial velocity u_0 and a free stream pressure p_0. Then, it flows across the disc with axial velocity u and pressure p_d^+ and p_d^- (before and after the rotor, respectively) as shown in Fig. 1. At the end, it leaves the control volume with an axial velocity u_1 and a far wake pressure p_1. For incompressible steady flows, Bernoulli's constant can be applied separately to a streamline both before and after the disc. Due to the energy extraction by the rotor, the total pressure or Bernoulli constant after the disc (H_1) remains constant but at a lower level than that of far upstream (H_0). The radial velocity before the rotor disc (w) is not zero because of flow expansion.

According to the experimental and numerical studies by Micallef et al. [13] and Herráez et al. [15], a significant radial flow in the rotor plane exists, especially in the outboard region. Based on the experimental and numerical results [12–15], the radial velocity after the disc is assumed to be the half of that before the disc. In the wake far downstream, the radial velocity is zero but the tangential velocity remains. Due to wake rotation, the static pressure p_1 is less than p_0, except at the boundary of control volume. Using the axial induction factors a and b (in the rotor plane and in the far wake, respectively) and the tangential induction factors a' and b' (in the rotor plane and in the far wake, respectively), the velocity at these locations is represented as

$$u = u_0(1-a),\ \mathrm{v} = 2\Omega a' r,\ u_1 = u_0(1-b),\ \mathrm{v}_1 = 2\Omega b' r \tag{1}$$

where Ω is the angular velocity of the rotor. The total pressure before the disc is

$$H_0 = \frac{1}{2}\rho u_0^2 + p_0 = \frac{1}{2}\rho u_0^2 (1-a)^2 + \frac{1}{2}\rho w^2 + p_d^+ \tag{2}$$

The total pressure after the disc is

$$\begin{aligned} H_1 &= \frac{1}{2}\rho u_0^2(1-a)^2 + \frac{1}{2}\rho(2\Omega a' r)^2 + \frac{1}{2}\rho\left(\frac{w}{2}\right)^2 + p_d^- \\ &= \frac{1}{2}\rho u_0^2(1-b)^2 + \frac{1}{2}\rho(2\Omega b' r_1)^2 + p_1 \end{aligned} \tag{3}$$

According to the conservation of angular momentum for inviscid annulus flows, that the following relation exists

$$a' r^2 = b' r_1^2 \tag{4}$$

Combing Equations (2)–(4), the pressure difference across the disc is obtained

$$\begin{aligned} \Delta p &= p_d^+ - p_d^- \\ &= \frac{1}{2}\rho u_0^2 (2-b)b + 2\rho(\Omega r)^2 a'(a'-b') - \frac{3}{8}\rho w^2 + p_0 - p_1 \end{aligned} \tag{5}$$

As the radial pressure gradient provides the radial equilibrium of the rotating wake, it is derived that

$$dp_1 = \rho(2\Omega b')^2 r_1 dr_1 \tag{6}$$

Integrating Equation (6), the pressure difference between far upstream and far downstream is

$$p_0 - p_1 = \int_{r_1}^{R_1} \rho(2\Omega b')^2 r_1 dr_1 \tag{7}$$

2.2. Axial induction in the far wake region

For incompressible and inviscid flows, the Euler equation in the non-inertial coordinates system rotating with the rotor can be written as