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Optimization of a thruster sections by Prandtl's theory

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A B S T R A C T

Wind power is now the world's fastest growing energy source. It is well known that energy production depends on the shape of the thruster (sail of yacht or a blade) and on the wind characteristics on a given site. The objective of this study is to maximize the performances of a thruster by optimizing the most important aerodynamic design parameters.

One of the most important challenges in aerodynamics is to accurately predict the forces acting on the thruster, where the thruster is modeled by different approaches such as the lifting line theory, where the application of the vortex filament method for the thruster performances is used. To have an optimal thruster, the elliptical lift distribution is the key design principle for the airfoils forming the thruster sections. The main purpose is to optimize the aerodynamic design of a thruster to increase its performances under wind action. The optimization carried out with computer codes for the design and analysis of a thruster, based on the non linear lifting line theory, shows that an optimal thruster is to be formed by airfoils placed along the span in a decreasing law of thickness.

1. Introduction

Among clean energy sources that are renewable, wind is regarded as less destructive to the environment. Wind power is currently one of the most reliable new energy sources known as a widely distributed clean and renewable source of energy. Due to the exponential growth of thrusters, and its general acceptance among people, in the past years, the trend in big propellers led to other design requirements. In fact, the researchers think always to enhance the accurate performances of thrusters. We will focus on thruster designs that require advanced technologies in: optimization both geometrically (aspect ratio, taper, twist …) and mechanically (thickness, camber …).

Thrusters might be divided in two categories: sails of yachts or blades of wind turbines. For the racing yachts, where the ship is not powered by an engine, many works have been carried out. A.D. Sneyd and T. Sugimoto [1] presented fundamental results concerning the optimum design of yacht sails and masts: the optimum design of yacht sail must be a compromise between the mast length, the moment and the heel angle. R. Nascimbene [2] developed multiple criteria optimization tools that allowed him to improve quality, efficiency and performances of the sail design. Also, the wind turbines have been studied by a lot of authors. A. Helali and al [3,4] have tried to solve the inverse optimum project for horizontal axis wind turbine in order to obtain the circulation distribution for a given extracted power and they have determined the chord lengths distribution law along the blade span. N. Brumioul [5] tried to maximize the energy production for a given site by optimizing the most important aerodynamic design parameters. In order to always be in coherence with reality, we have tried to simplify the thruster model (sail or blade), so it depends only on a few simple design parameters with broad applicability.

The analysis and the optimization of numerical methods have long been of a major role in the design of thrusters, in both for the industry and for the university researchers [6–8]. In fact, the aerodynamic characteristics of the thruster are closely related to its geometry. Many studies using the lifting line theory, approximating the thruster and its wake by a cluster of horse-shoe vortices have been carried out. Several attempts have been made to look for optimum thruster designs. Our aim is simply to numerically calculate the ideal circulation distribution of the thruster.

The classic lifting line theory was developed by Ludwig Prandtl and Max Munk in Göttingen, Germany [9]. It proposes a simple modeling of finite span thruster placed perpendicularly to the flow: the thruster is modeled by a system of horseshoe vortices of infinitesimal intensity superimposed on the line of the trailing

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Nomenclature

AR:	Aspect ratio
b:	Thruster length
c:	Chord
c_t:	Tip chord
c_r:	Root chord
c_l:	Sectional lift coefficient
C_L:	Lift coefficient
C_{Di}:	Induced drag
$e_{\max}$:	Max thickness
K:	Constant
$L_{\max}$:	Max camber
V_∞:	Free stream velocity
α:	Geometric incidence
α_{eff}:	Effective incidence
α_i:	Induced incidence
α_v:	Twist incidence
λ:	Taper ratio
Γ:	Circulation

edge. Based on this theory, WF Phillips and DO Snyder [10] have developed a numerical model based on a fully three-dimensional vortex lifting law, that can be used for systems of lifting surfaces with arbitrary camber, sweep, and dihedral. This model gives results with accuracy comparable to that obtained from numerical panel methods and inviscid computational fluid dynamics solutions. Similarly, based on the classic lifting line theory, and on experimental tests, A. Merabet [11] showed that the aerodynamic properties of shape wings arbitrary plane are not radically different from the optimal wing: the elliptical. Although several authors have studied the effect of aspect ratio on the performances of the thruster, few studies have been considered in analyzing the twist and taper effect. Our work is to present and discuss the effect of these parameters on the lift, and thus provide a design topology for the thruster shape. This part of the study is based on the classical lifting line theory opting for the Fourier series development to resolve the fundamental equation of Prandtl's.

Seen that the thruster geometry optimization is not satisfactory, the definition of the airfoils forming the thruster is also recommended. The non-linear lifting line theory appears to be one of the appropriate solutions for the treatment of this problem. This theory, which is an extension of the classical theory, takes into account the nonlinear regime of the lift curve. It is based on the assumption that the lift generated by each thruster section circulation can be assimilated to the lift generated by a similar 2D airfoil. Anderson [12] mentions that if we know the data of the 2D airfoil above stall, we may well get a solution using an algorithm based on the non-linear lifting line theory. Hunsaker [13] validated numerically the statement of Anderson. In addition, Philips [14] proposed a modified numerical lifting line method that can converge to a wing above stall. Hui Liang and al [15] Robert E. Spall and al [16] and Hunsaker [13] have used nonlinear theory respectively to make an unsteady study, optimize the geometry and shape of the wing, and consider the stall. These researchers have handled only the case of wings formed by a single type of airfoil. One of the main objectives in the present paper is to pursue Anderson's work [12] in the sense of studying a thruster formed by different types of airfoils, by defining their optimal locations along the span.

Two numerical models based on the two theories of Prandtl were developed in FORTRAN language. A comparison of the results of the two calculation codes with data from the literature [12,13] shows that the two numerical models give reasonable results close to the experimental.

2. Methodology

2.1. Classic lifting line theory

According to the lifting line theory [17], the law of circulation along the blade is expressed as:

$$\Gamma(y) = \frac{1}{2} c(y) \cdot C_L \cdot V_\infty \quad (1)$$

where C_L the lift coefficient for a section y, it is given by:

$$C_L = K \cdot \alpha_{eff} \quad (2)$$

With α_{eff} is the incidence actually seen by the airfoil: it is the algebraic sum of the geometric incidence $\alpha(y)$, the twist incidence $\alpha_v(y)$ and the induced incidence $\alpha_i(y)$.

K is the curve slope and C_L is a function of the incidence.

Let's denote: In the case of flat plate airfoil, $K = 2\pi$.

Hence, by replacing α_{eff} by its expression, the fundamental equation of Prandtl takes the form [12]:

$$\Gamma(y) = \pi \cdot c(y) \cdot V_\infty [\alpha(y) + \alpha_v(y) + \alpha_i(y)] \quad (3)$$

The fundamental equation of Prandtl (3) can be solved either by the method of Glauert–Carafoli [18], or by a Fourier series development of the circulation [19].

The Glauert–Carafoli gives a solution for the aerodynamic properties of a finite thruster with elliptical lift distribution; however, the Fourier series development is able to give a solution for a finite thruster with general lift distribution. As we are dealing in this paper with thrusters having different geometries and lift distributions, our numerical simulation is based on the Fourier series development.

- General lift distribution

Consider the transformation

$$y = -\frac{b}{2} \cos\theta \quad (4)$$

The coordinate in the direction of the span is now given by θ, where $0 \le \theta \le \pi$.

In terms of θ, the elliptical circulation distribution can be written as follows

$$\Gamma(\theta) = \Gamma_0 \sin\theta \quad (5)$$

This equation shows that the use of a Fourier sine series would be an appropriate expression for the distribution of a general circulation along an arbitrary finite thruster:

$$\Gamma(\theta) = 2bV_\infty \sum_{1}^{N} A_n \sin n\theta \quad (6)$$

where N must be as high as possible for accuracy and A_n should satisfy the fundamental equation of Prandtl's lifting line theory.

By substituting the Fourier sine series in equation (3) we obtain:

$$\alpha(\theta_0) = \frac{2b}{\pi c(\theta_0)} \sum_{1}^{N} A_n \sin n\theta_0 + \sum_{1}^{N} nA_n \frac{\sin n\theta_0}{\sin\theta_0} \quad (7)$$

The equation (7) is evaluated at a given spanwise location θ_0;

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