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Transient mechanical wave propagation in semi-infinite porous media using a finite element approach

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Abstract

In this study, we propose a numerical investigation in the time domain of the mechanical wave propagation of an impulsional load on semi-infinite soil. The ground is modelled as a porous saturated viscoelastic medium involving complete Biot theory. All the couplings and a hysteretic Rayleigh damping are taken into consideration. An accurate and efficient finite element method using a matrix-free technique and an expert multigrid system are applied. Our results present the displacements of the fluid and solid particles over the surface and in depth. The arrival times of body and surface waves are studied. Particularly, the compressional wave of the second kind is highlighted. The influence of the different couplings and more specifically, the influence of the permeability on the response of the soil are analyzed.

Keywords: Time domain; Wave propagation; Porous media; Finite element; Biot theory; Second compressional wave

1. Introduction

The study of the mechanical wave propagation in a transient regime in semi-infinite saturated porous soil is a problem of great importance in a large number of areas of applied mechanics and geomechanics ranging from earthquake engineering to soil vibrations or soil–structure interactions.

A saturated porous medium is a medium that presents on the microscopic spatial scale a solid part and a porous space filled with a viscous fluid. When we focus our attention on the description of such a medium, two approaches are possible. The first approach is situated at microscopic scale. In this configuration, the 'solid elastic' phase and the 'compressible viscous fluid' phase each constitute distinct geometric domains. A geometric point is found, at a given instant, in one of these two clearly identifiable phases. The second approach looks at the problem from the macroscopic level. The elementary volume is considered to be the superposition of two material particles occupying the same geometric points at the same instants with different kinematics. Thus, the saturated porous medium is considered as a two-phase continuum: the skeleton particle is constituted by the solid matrix and the connected porous space, and the fluid particle is formed from the fluid saturating this connected porous space.

The change of microscopic–macroscopic scale has been notably studied by Auriault [1], Burridge and Keller [2], Terada et al. [3] and by Coussy et al. [4]. These authors study the solid–fluid mixture. Approaching the problem from the scale of the pore, they formulate the mechanical equations relevant to each phase and the mechanical equations relevant to the couplings of the mixture. Homogenization is then obtained through asymptotic developments or mathematical averaging procedures.

Biot's equations which macroscopically govern the two-phase coupled porous medium are chronologically anterior to previous studies and have been subsequently justified. The macroscopic coefficients take their physical meaning in part from the microscopic characteristics of the medium. Biot's two articles [5,6] are works of reference for mechanical wave propagation theory. The two articles of 1962 [7,8] aim towards a more general reformulation of previous studies including anisotropy, viscoelasticity and internal dissipation of the medium. Zienkiewicz et al. [9]

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focus on the peripheral cases of Biot's model for drained and undrained soil or in the quasi-static case. The different internal physical damping of the soil corresponding with the viscoelastic character of the skeleton particle are synthesized by Lefeuve-Mesgouez [10].

Different kinds of body waves exist in a porous medium: two compressional waves and a shear wave S. The first compressional wave P1 is a wave said to be quick whereas the second compressional wave P2 is said to be slow and attenuated. Additionally, should the medium present a free surface, a Rayleigh surface wave R will appear.

The validity of Biot's model and especially the experimental observation of the P2 wave have been obtained by Plona [11] and Berryman [12].

In the transient regime specialist area, Ghaboussi and Wilson [13] are the first to propose a numerical approach using finite elements based on Biot's model. However, this research does not take the notion of tortuosity into account, it artificially introduces a structural damping and gives a few results in 1D. Prevost [14] also presents theoretically and then numerically some 1D and 2D results, essentially in the quasi-static case. Afterwards, Zienkiewicz and Shiomi [15] present synthetically a finite element formulation $\{u, p, U\}$ of Biot's equations without tortuosity. Three formulations are studied and compared: first, an exact formulation $\{u, U\}$ for a compressible fluid; second, an approximate formulation $\{u, U\}$ for an incompressible fluid, for which the pressure term is approached using the Penalty Method; and finally an approximate formulation $\{u, p\}$ for which the fluid acceleration term is neglected. The authors underscore the large numerical oscillations for the exact formulation $\{u, U\}$ in 2D which can be reduced through the artificial introduction of a numerical damping.

Simon et al. [16,17] present a summary work on the diverse existing finite element formulations and the different techniques of time resolution. Their objective is to study the precision of the results by comparing these approaches to a theoretical approach presented in [18]. The study is carried out in 1D, using a hypothesis of dynamic compatibility and the Biot model is still without tortuosity.

In the case of an incompressible fluid and an incompressible solid, Gajo et al. [19] directly resolve the $\{u, p, U\}$ formulation. The authors essentially present their results in 1D and compare them with an analytical solution, as described in [20].

Other research such as that carried out by Hörlin et al. [21], Dauchez et al. [22,23], Atalla et al. [24] or Göransson [25] proposes finite element formulations in a permanent regime based on Biot's equations in $\{u-U\}$, $\{u-p\}$ or $\{u-\psi\}$ where ψ is a fluid potential.

In this article, we propose an accurate and efficient finite element $\{u-U\}$ formulation of Biot's equations in a transient regime. Thus, this time domain approach allows us to accede to a complementary understanding of the signal, for instance in the determination of the wave propagation celerities. The problem that we focus on here concerns a two-dimensional saturated porous semi-infinite medium subjected to an impulsional excitation. Biot's equations are written in their complete, dimensionless form. All couplings (mass, inertial and elastic) are thereby taken into account. The finite element formulation includes a Rayleigh hysteretic viscoelastic damping. The objective of the article is to visualize the propagation of the different waves over the surface and in depth for the half-space in the time domain. Specifically, the P2 wave is given prominence. We analyze the influence of the couplings on the response of the displacements in the medium. To our knowledge, this particular approach has not been previously proposed.

2. Field equations

The macroscopic equations for dynamic isotropic saturated poroelasticity for small strains in a Lagrangian description were first formulated by Biot [5,6]. Bourbié et al. [26] proposed a complete review of the Biot theory.

The first equation of motion for the global system without body force can be written as

$$\sigma_{ij,j} = (1-\phi)\rho_s\ddot{u}_i + \phi\rho_f\ddot{U}_i \tag{1}$$

In the above equation, u_i and U_i, respectively, represent the displacement components of the skeleton particle and the fluid particle, σ_{ij} the total Cauchy stress tensor components, ϕ the porosity defined by the connected space where fluid flow occurs over the elementary volume and ρ_s and ρ_f respectively the densities of the solid grains and the fluid component. The subscripts $()_{,i}$ and the superscripts $(\cdot)$ each denote respectively spatial and time derivatives. The summation convention is applied.

A second equation of motion that corresponds with a generalized law of Darcy in transient regimes can be written in the following form

$$p_{,i} = -\frac{\phi}{K}(\dot{U}_i - \dot{u}_i) + \rho_f(a-1)\ddot{u}_i - a\rho_f\ddot{U}_i \tag{2}$$

where p is the pore pressure in the fluid, K the hydraulic permeability coefficient defined by the absolute permeability coefficient over the dynamic viscosity of the fluid which represents the viscous coupling and a the tortuosity coefficient which represents the inertial coupling.

The two constitutive relationships between stress and strain can be written as follows

$$\sigma_{ij} = \lambda_{0v}\varepsilon_{kk}\delta_{ij} + 2\mu_v\varepsilon_{ij} - \beta p\delta_{ij} \tag{3}$$

$$-\phi(U_{k,k} - u_{k,k}) = \beta u_{k,k} + \frac{1}{M}p \tag{4}$$

in which M is the first Biot coefficient, β the second Biot coefficient representing the elastic coupling and δ_{ij} the Kronecker symbol; $\varepsilon_{ij}=1/2(u_{i,j}+u_{j,i})$ is the strain tensor component of the skeleton particle in the case of small deformations and $U_{i,i}$ the fluid dilatation. The Biot